PHILOSOPHICAL

TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXVII.

PART II.

LONDON,

PRINTED BY W BULMER AND CO. CLEVELAND-ROW, ST. JAMES'S;
AND SOLD BY G. AND W. NICOL, PALL-MALL, BOOKSELLERS TO HIS MAJESTY,
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PHILOSOPHICAL TRANSACTIONS.

XIII. *Description of a thermometrical barometer for measuring altitudes. By the Rev.* Francis John Hyde Wollaston, *B. D. F. R. S.*

Read March 6, 1817.

HAVING had my attention drawn, some years ago, when engaged in experimental Lectures at Cambridge, to the variations in the heat of boiling water as corresponding with the changes in atmospherical pressure and the height of the barometer, I constructed several very sensible thermometers, for the purpose of ascertaining these variations with minuteness, and have been led on by my observations into making an instrument, which I believe may be useful in measuring heights with greater accuracy and convenience than the common barometer. This is not proposed as a new idea, for I find that FAHRENHEIT has suggested it in his "Barometri novi descriptio," Phil. Trans. Vol. 33, p. 179, and also Mr. CAVALLO, Phil. Trans. Vol. 71, p. 524. But the instrument is carried farther than had been done by them.

At first my thermometers were made with different scales

from the uncertainty how far sensibility might be carried in them. In one instance my thread was so fine, and I had made so large a bulb, that every degree on FAHRENHEIT's scale was equal to 10 inches, and by connecting different threads to different bulbs, I have had them of all varieties from that length to half an inch.

The instrument, with which I have made the greatest number of observations, has a scale of 3,98 inches to every degree; the thread, which is 22 inches long, was proved before its attachment to the bulb, and being found not cylindrical, the proper allowance has been every where made for the variations in the different parts of it. The degree was ascertained by comparison with a good thermometer at low temperature before closing the tube. The degrees I divided into 100 parts on the scale, and by a Vernier into 1000. This has been compared with a common barometer, the height being always corrected for temperature according to General ROY's Table in Phil. Trans. Vol. 67, p. 687. With this its agreement has been very close, after I had detected by means of it and had corrected two inaccuracies in my barometer, which would otherwise have escaped me. One was in the total length from the basin to the scale, and was ascertained by a comparison made by means of a thermometer between my own barometer and two excellent mountain barometers, by TROUGHTON and CARY. The other appeared by a want of agreement at low barometrical heights, when they agreed well above. For this I was at a loss to account, till I conjectured that it might arise from the greater quantity of mercury then expelled from the tube of the barometer and rising into the upper part of the basin, where the wooden box from

being thicker would occasion a greater variation. By altering the quantity of mercury in the basin I removed this cause of inaccuracy, and the instruments have agreed equally well in all parts between 30,68 on barometer and 28,23. The result of the comparison is, that a difference of 1° FAHRENHEIT is occasioned by 0,589 on the barometer corrected. 30,603 corrected barometer = 213,367 thermometer, and 28,191 barometer = 209,263 thermometer. There will be variations from this general result when the difference below the mean heights is considerable; but I did not attempt the observation of them, as my barometer is not provided with an adjustment of the mercury in the basin, and this thermometer is, I think, of too long a scale.

By these trials being satisfied of the capability of the instrument, I have endeavoured to render it as portable as possible, for the farther purpose of measuring difference of altitudes, of which I found it very sensible; and will describe what I may call, following FAHRENHEIT and CAVALLO, a thermometrical barometer, and may venture to recommend for use.

Pl. VII. Fig. 1. represents the thermometer. The bulb A, one inch diameter, is blown thick and strong on a tube of thick glass, the bore of which is not material, say $\frac{1}{40}$ inch. It is better to make the bulb on a separate thick tube, and to join the fine thread afterwards, than to attempt to blow it sufficiently large and strong through the fine thread itself. The thickness and strength of the bulb in every part is essential to its not yielding. Close above the bulb a swell B is made to contain, *as near as may be*, whatever mercury expands out of the bulb between the common temperature and that of boiling water. If this be too small, the mercury contracts into

the bulb, and may change its place by a shake; if too large, a part of the mercury remaining in B may be detached by a shake, and occasion great inconvenience. To prevent that possible detachment, the long shape given in the figure is preferable to a spherical swell. A workman accustomed to blowing thermometers, though he will at first make it too large, will soon hit the size. When the metal is hot in this part, a slight pressure of the tube endwise will occasion a little thickening of the glass externally about the part C, which is of use for fixing the thermometer in its mounting.

For the fine thread, the tube D is chosen by comparison with other thermometers, such that if a bulb were blown on it of 0,4 inch in diameter, its scale would be about four inches between freezing and blood heat; that is, 16° to an inch; when this tube is fitted to a bulb of an inch diameter, its degrees will therefore be about an inch each. The tube is five inches long. Before the joining is made at E, the bulb is filled: and the upper end of D being broken off *abruptly* is joined by its edges to a small piece of tube F of the same external diameter, but of an open bore, so as to make a sort of bulb at the top by the cap F; a blown bulb will not answer the purpose for which it is designed, of detaching a globule of mercury from the thread, and retaining it apart for future use. The joining at E must be neatly made without any crevices in which either mercury or air may lodge, and with as little swell in the thread as possible; for if there be any thing that can be called bulb in that part which is protected by the mounting from the action of the heat, the thermometer will be long in feeling its whole expansion.

Before F is sealed, the bulb and swell and F together con-

taining an excess of mercury, boil the thermometer in water, and if the instrument be wanted for the measurement of a height of 5000 feet, let the whole cool down to 200; if for 10,000 feet let it cool to 190, drawing the mercury into the thread: and at that point hastily tilt off the mercury remaining in F, which may then be sealed while the whole is kept hot. On boiling it up again, the excess of mercury which rises into F will be detached from the thread by a gentle tap or two on the side with the nail, and will remain in the cap F for use when required.

For mounting the thermometer, GH, Fig. 2. is a circular plate, one inch in diameter, with a hollow half cylinder K rivetted firmly through it, of sufficient size to admit the lower tube of the thermometer to be bound firmly to it, so that there be no shake, and no reliance for steadiness on the more tender part of the tube above. The hole L fits the tube pretty closely.

Fig. 3. is a second circular plate 1,5 in diameter with two screw collars 1,15 diameter of the same external thread, and a hole M in its centre for the tube of the thermometer to pass through it. The holes L and M being opened conically in opposite directions, allow a little fine tow to be wound round the tube, and when the two plates are fixed together by screws passing through them, they close the tow round the tube, both to steady it and to prevent the escape of steam.

Fig. 4, is the scale, 5 inches long, 0,9 broad; between the two standards N O a length of 4,15 inches is divided into 100 parts, and by the Vernier into 1000, giving 241 parts to the inch; this was accidental, being occasioned originally by the thread of my adjusting screw, which assisted me in making the divisions. The scale is fixed down to the upper plate,

Fig. 3, within the collar, by screws passing through the flanch P, at the back of the scale. It would be well, that a piece of thick leather, or soft wood, should be screwed between these two, if it can be done with sufficient firmness, for the purpose of preventing the scale getting inconveniently hot.

The adjusting screw, which carries the Vernier, is raised by the standards above the scale, and is placed opposite the centres of the plates in fig. 2 and 3, by which means the milled head Q goes better into the case hereafter described. The tube of the thermometer, when passed through the central holes in the two plates, turns by its bend to the left hand, and up the side of the scale, being slightly fastened to it at the top only, with a small piece of cork under it, to keep it clear from the scale. The Vernier has fixed to it, by means of a screw head, two pieces of thick paper laid upon each other, the one black and the other white, half of the outer paper being cut away straight, makes a line between the black and white, better than any that can be drawn for adjustment to the top of the mercurial thread.

Were I wanting another of these instruments, and employing a workman to construct it for me, I would have the whole length of the adjusting rod square, with a small piece sliding by hand on it for the larger alterations, and carrying a short screw for the finer movements, or the whole scale and movements might, I think, conveniently be made with tubes in the manner of the mountain barometers, and the thermometer would not in that case require the bend, but would run up the centre.

To the Vernier Mr. CARY has attached for me, with a joint, a small lens of one inch focus, which assists in observing the

height of the mercurial thread, and by having no lateral motion, confines the view to the same direction and prevents parallax.

In boiling, the bulb should be exposed to steam only, as being steadier in its heat than the water. My boiler is a tin cylinder 5,5 deep, 1,2 diameter, with an external cylinder 1,4 diameter for preventing the transmission of heat, the bottom only is single. The interior cylinder has a brass collar soldered into it, having an internal screw which fits to either of the external screws on the plate Fig. 3.; so that what is boiler when fixed below plate 3, becomes a case to protect the scale, when screwed to the upper side of that plate. The top of the external cylinder being closed into the same brass collar, becomes slightly conical, and is soldered to it. An opening of 0.2 diameter is made through both cylinders, immediately under the collar, as a vent for the steam from within, but is prevented from communicating with the annular space between the vessels, lest inconvenience should arise from water accidentally getting between them.

Another tin cylinder, 1,2 in diameter, and 2,1 deep, with a similar screw collar at top, forms a case for protecting the bulb when screwed to the under side of plate 3, and is also a measure for the quantity of water put into the boiler, which should not touch the bulb; it is here 1,25 below it.

For the purpose of rendering every thing requisite for use portable also with the thermometer, I made a stand for it, which is convenient and will readily be described. Round the outside of the boiler, and just below the conical closing of it, is soldered a ring of brass wire ST. Fitting to the conical top of the boiler is made another short cone of thick tin, which

may be fastened down by screwing the thermometer into the boiler; or, which is better, by a separate collar U for the purpose, to screw into the boiler, having the same internal screw above to receive the thermometer. This conical cap has a wire soldered round it at VWX, and on this wire turn by eyes at their ends seven wires, nine inches long and of sufficient strength. They are placed at six equal distances round the cap, two of them being placed close together. These wires by bearing on the ring ST are spread outward, and being connected by gores of thin linen, sewed between them from top to bottom, are prevented from spreading beyond a certain point and form a very steady base for the support of the whole instrument, and at the same time a bell tent to protect the lamp and boiler from the wind. The two wire legs, which are placed together, are not connected otherwise than by a hook at bottom, and will allow the tent to be opened at that part for examining and adjusting the lamp, while the instrument stands firm on the remaining legs. The lamp (fig. 6.) is a cylinder 1,8 in diameter, and 0,9 deep, having a tube in the centre to carry a wick, and a cover with six holes round it of 0,8 diameter each, and an opening in the middle 0,8 diameter; a copper pipe, 0,85 in diameter, and 1,1 long, turns over the opening by means of a hinge, and forms a chimney to prevent smoke, on the principle of ARGAND's lamp. I burn oil with a quantity of tallow added, to make it congeal. The lamp has a rod of strong wire fixed to its circumference within, and sliding in a tube YZ on the outside of the boiler. Fig. 7. represents the instrument on its stand for use.

To pack the instrument for carriage, the thermometer is secured by being screwed into its upper and under caps, and

is then enveloped with the bulb downward in the folds of the tent inverted. The lamp is put first into the case, and the other parts being thrust down afterwards, are kept very steady by the linen of the tent. The whole goes into a tin cylindrical box, two inches diameter and ten deep, and weighs 1lb. $4\frac{1}{2}$oz.

The scale of an inch to a degree is chosen, because on trying various threads, I have found that when extremely fine, it is almost impossible to give such strength to the bulb as to force the column of mercury accurately to the same height on repetition of the boiling, by reason of the resistance from friction in the tube. With an inch scale, the variations of the barometrical thermometer will be to those of the common barometer as 5 to 3, and the sensibility in this instrument is so fully sufficient, that the difference of temperature arising from the height of a common table is immediately perceptible. If more were wanted, either the thread must be finer, which would endanger the precision; or the bulb larger, the objections to which are obvious. Adhesion to the glass and friction must always have some small effect: and for the same reason that a common barometer is shaken on observation, this instrument, when boiled, should be tapped gently two or three times on the side, to free the mercury; when that is done, whether on the rising of the column to its height, or the falling of it after a forced expansion, it will with this sized thread come to the same place precisely.

Upon trying this thermometer when mounted, I found that a variation of 0,589 barometer, which was before ascertained = 1 Fahrenheit, would be equal to 283 parts on my scale = 0,97 inch; and an inch on barometer would produce a

variation of 395 parts, or 1,648 inch on thermometrical barometer. My whole scale of 1000 parts would therefore equal 2,52 of barometer, and comprehend all changes from 28,1 to 30,6, if wanted to compare its variations with those of a common barometer. Having observed also with a former thermometer mounted on this scale, that 1° Fahrenheit or 0,589 barometer, was equal to an elevation of about 530 feet, I reckon that as on my thermometrical barometer 500 parts from the top would correspond with 29,8 barometer, I have at any height from 29,8 upwards, the other 500 parts or more to be applied to the measurement of altitudes, which allowing 283 parts to 530 feet, will comprehend every thing that can be wanted in this part of the kingdom. This being convenient for all my purposes, the thermometer was to be set to this point. I therefore first drew all the mercury from the cap F into the tube, and then expanding it carefully and gradually out, shook off the globules at top, till it should stand on boiling at such height according to the state of the barometer at the time, that by computation the top and bottom of the scale should correspond with 30,6 and 28,1 respectively. It is for the shaking off this globule that the fine tube must be broken abruptly at top and end flat; and there is no fear of the globule being drawn down again into the tube, unless the whole column is expanded to the top and connected with the globule in F.

Yet this instrument, though adjusted now to my own particular use by the quantity of mercury in the thread, is capable of measuring any greater altitude, even Mt. Blanc or Chimboraço, under any barometrical circumstances, and the change for that purpose is much easier effected than would

be imagined. At the lowest station (having previously drawn into the tube the whole globule in F by expanding the mercury and connecting the thread at top) boil the thermometer; put a wooden peg into the steam vent, and by that forced expansion shake off a few parts at the top of the scale, so that the boiling point may be there accurately taken. Ascend till on boiling again the point is near the bottom of the scale, which will be with this particular instrument about 2200 feet. After an observation at this second station, adapt the thermometrical barometer to measuring in like manner another ascent to a third station, by drawing the globule in again, and expanding out the excess as before, so that the mercury may stand again at the top of the scale, and its point may be noted. Hence proceed to a fourth station, and so in like manner to others, getting the difference between the several stations, and consequently the aggregate height. The only correction requisite is for the specific gravity of air at different temperatures given by General Roy, Phil. Trans. 67, 770, for this a small thermometer is wanted, and will find room in the case among the folds of the linen tent.

The experiments, which I have been able to make on altitudes, have been few, and they were made with a different instrument, which was unluckily broken. It had more sensibility than that which I have described, 1° Fahrenheit being equal to 552 parts on my scale, or 2,3 inches. By a few observations, such as the height of my house afforded, I reckoned 552 parts equal to 530 feet in altitude. With that instrument, boiled on the counter of a bookseller's shop in Paternoster-row, which I estimated between four and five feet

above the foot pavement on the north side of St. Paul's church-yard, and boiled again in the Gilt Gallery, there was a difference of 254 parts, barometer being 29,92, thermometer 77. 552 parts : 530 feet :: 254 parts : 243,87 feet, to which add the correction by Roy's table $\frac{118}{1000}$ of the height = 28.77, and the corrected height is 272.64. General Roy makes the gallery above the north pavement to be 281 feet, which allowing 4 or 5 feet for the difference of our lower stations, would give 266 or 267 feet for my observed height, differing only by about 4 feet. If I take my proportion from Roy's statement that 1° Fahrenheit = 535 feet, the result will be still nearer. 552 : 535 :: 254 : 246,1, add correction $\frac{118}{1000}$ = 29, the corrected altitude will be 275,1 feet, differing less than 2 feet.

One other observation I made with the same thermometer on a height ascertained also by General Roy. My thermometer boiled at the ferry-house, opposite Woolwich arsenal, stood at 869, and in the Prospect-room at the Inn on Shooter's hill, it showed 432, a difference of 437 parts, barometer 29,94, thermometer 58.

552 : 530 :: 432 : 419,6
Add correction $\frac{67.5}{1000}$ = 28,3 } height corrected = 447.9.

By Roy, the height from the Gun wharf at Woolwich to the upper story at Shooter's hill is 444 feet.

If this instrument should be thought deserving of being brought into use for scientific purposes, the sensibility and length of the thermometer would be selected by each person according to the particular object in view. With a sensibility equal to that of a common barometer, 1° of Fahrenheit

being equal to 0,589 inch, whether a scale of the common or thermometrical barometer, an equal precision may, I think, be attained; and with a power considerably increased, the instrument will always be much more portable than a common barometer, although the scale should be extended for taking the greatest known heights by a single pair of observations.

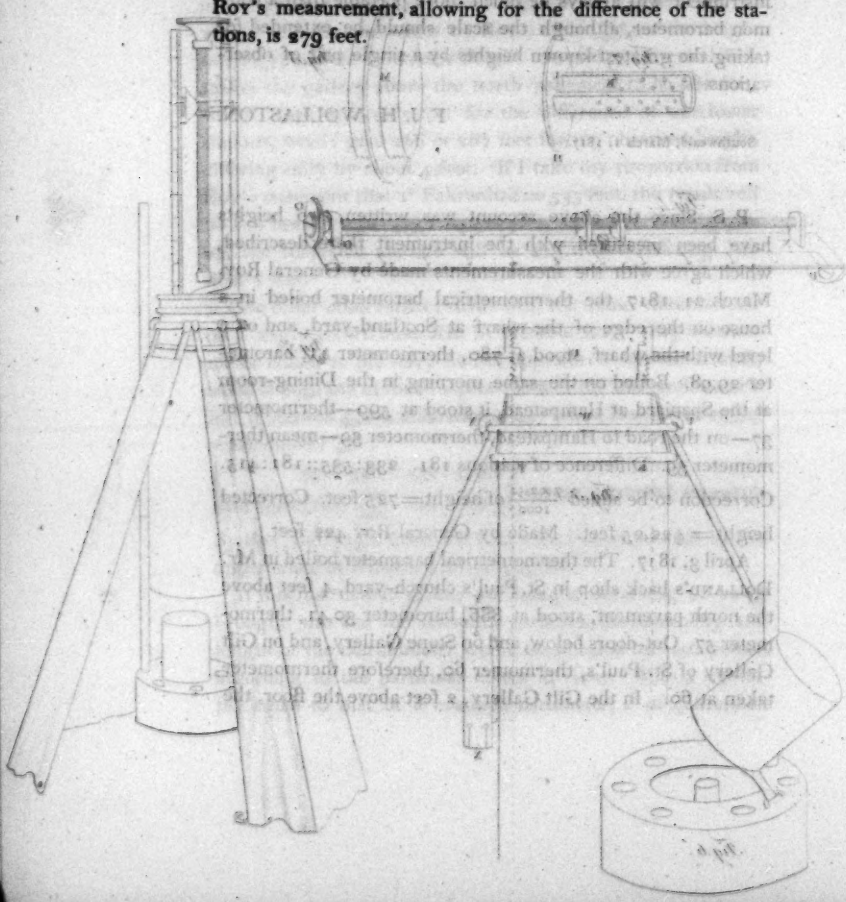
F. J. H. WOLLASTON.

Southwould, March 1, 1817.

P. S. Since the above account was written, two heights have been measured with the instrument there described, which agree with the measurements made by General Roy. March 21, 1817, the thermometrical barometer boiled in a house on the edge of the wharf at Scotland-yard, and on a level with the wharf, stood at 780, thermometer 41, barometer 29,98. Boiled on the same morning in the Dining-room at the Spaniard at Hampstead, it stood at 599—thermometer 37—on the road to Hampstead, thermometer 39—mean thermometer 39. Difference of stations 181. 938 : 535 :: 181 : 415. Correction to be added $\frac{7 \times 2,52}{1000}$ of height = 725 feet. Corrected height = 422,25 feet. Made by General Roy 422 feet.

April 3, 1817. The thermometrical barometer boiled in Mr. Dolland's back shop in St. Paul's church-yard, 4 feet above the north pavement, stood at 886, barometer 30,41, thermometer 57. Out-doors below, and on Stone Gallery, and on Gilt Gallery of St. Paul's, thermomer 60, therefore thermometer taken at 60. In the Gilt Gallery, 2 feet above the floor, the

thermometrical barometer stood at 778,5. Difference = 112,4. 288 : 585 :: 112,4 : 258. Correction = $\frac{74,5}{1000}$ of height = 19,2 feet. Corrected height = 258 + 19,2 = 277,2 feet. General ROY's measurement, allowing for the difference of the stations, is 279 feet.



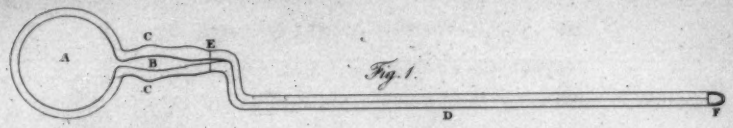


Fig. 1.

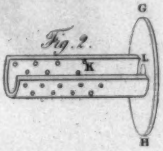


Fig. 2.

Fig. 3.

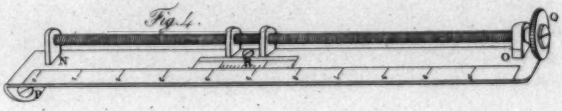


Fig. 4.

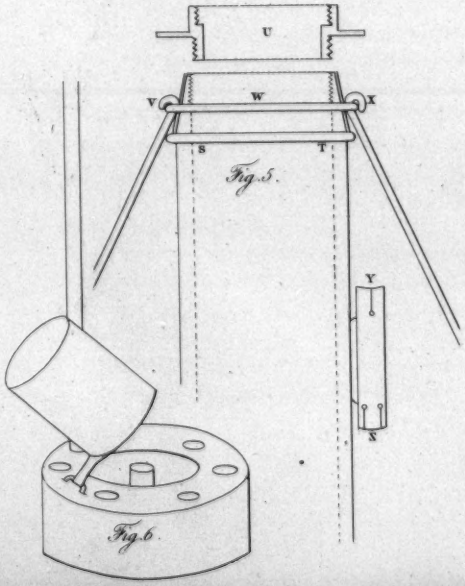


Fig. 5.

Fig. 6.

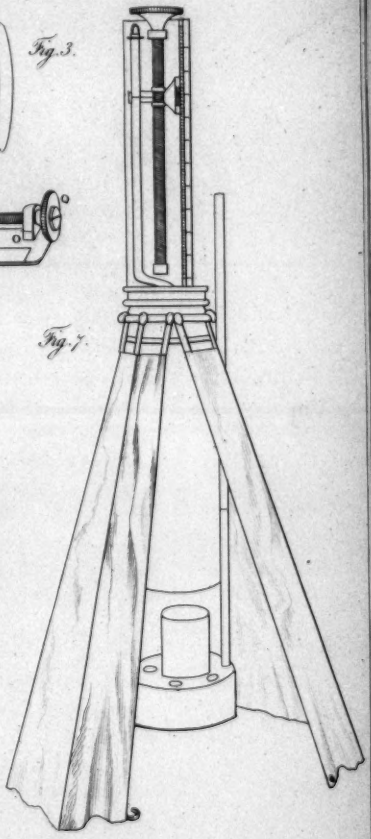


Fig. 7.

XIV. *Observations on the Analogy which subsists between the Calculus of Functions and other branches of Analysis.* By Charles Babbage, *Esq. M. A. F. R. S.*

Read April 17, 1817.

It is my intention in the following Paper to offer to the Royal Society some remarks on the utility of analogical reasoning in mathematical subjects, and to illustrate them by some striking facts which have occurred to me, when comparing the calculus of functions with other modes of calculation with which mathematicians have been long acquainted. The employment of such an instrument may, perhaps, create surprise in those who have been accustomed to view this science as one which is founded on the most perfect demonstration, and it may be imagined that the vagueness and errors which *analogy*, when unskilfully employed, has sometimes introduced into other sciences, would be transferred to this.

It is, however, only as a guide to point out the road to discovery, that analogy should be used, and for this purpose it is admirably adapted.

It is usually more difficult to discover than to demonstrate any proposition; for the latter process we may have rules, but for the former we have none. The traces of those ideas which, in the mind of the discoverer of any new truth, connect the unknown with the known, are so faint, and his attention is so much more intensely directed to the object, than to

the means by which he attains it, that it not unfrequently happens, that while we admire the happiness of a discovery, we are totally at a loss to conceive the steps by which its author ascended to it.

From these considerations, I think it will appear, that any successful attempt to embody into language those fleeting laws by which the genius of the inventor is insensibly guided in the exercise of the most splendid privilege of intellect, would contribute more to the future progress of mathematical science than any thing which has hitherto been accomplished. Amidst the total absence of all attempts of this kind, the following illustrations of one of the most obvious assistants of the inventive faculty, will not, I hope, be considered useless, even though it should have no other effect than that of directing the attention of those who are engaged in mathematical enquiries, to this most interesting and important subject.

At our first entrance into algebra, one of the most remarkable circumstances which presents itself is, that some fractions which in certain cases apparently vanish, have in fact a finite value. Such is the fraction $\frac{a^x - b^x}{x}$ which when $x = o$ becomes $\frac{1-1}{o} = \frac{o}{o}$, and yet its real value is well known to be $\log. \frac{a}{b}$.

Here then by assigning a certain value to a variable quantity which is capable of all degrees of *magnitude*, the expression apparently becomes illusory: let us now examine a parallel case in the Calculus of Functions. Take the expression $\frac{\psi x - \psi \frac{1}{x}}{\psi x}$, and let us suppose ψ to be an arbitrary charac-

teristic capable of assuming all varieties of *form*; then amongst these varieties we may have $\psi x = o$, and the expression becomes $\frac{o-o}{o}$.

In order to ascertain its real meaning, let us suppose

$$\psi x = v\varphi x + v^2_2\varphi x + \&c.$$

which if we suppose $v = o$ gives $\psi x = o$

then $$\psi \tfrac{1}{x} = v\varphi \tfrac{1}{x} + v^2_2\varphi \tfrac{1}{x} + \&c.$$

and the expression becomes

$$\frac{\psi x - \psi \frac{1}{x}}{\psi x} = \frac{\left(\varphi x - \varphi \frac{1}{x}\right)v + \left(\varphi_2 x - \varphi_2 \frac{1}{x}\right)v^2 + \&c.}{v\varphi x + v^2_2\varphi x + \&c.} = \frac{\left(\varphi x - \varphi \frac{1}{x}\right) + \left(\varphi_2 x - \varphi_2 \frac{1}{x}\right)v + \&c.}{\varphi x + v\varphi_2 x + \&c.}$$

which becomes when $v = o$ or $\psi x = o$

$$\frac{\psi x - \psi \frac{1}{x}}{\psi x} = \frac{\varphi x - \varphi \frac{1}{x}}{\varphi x}$$

where φx is quite arbitrary.

If we suppose ψx to become a symmetrical function of x and $\frac{1}{x}$, we have $\psi x = \psi \frac{1}{x}$ and our expression becomes $\frac{o}{\psi x}$ which actually does vanish in all cases except at the same time $\psi x = o$.

As a second example take the expression $\frac{fx - fx \,.\, f_2 \alpha x}{1 - fx \,.\, f\alpha x}$ which becomes $\frac{o}{o}$ if the two following equations hold true $fx \,.\, f\alpha x = 1$ and $fx - fx \,.\, f_2 \alpha x = o$, α being any given function, such that $\alpha^2 x = x$. In order to ascertain its real value in that case, let us suppose

$$fx \text{ becomes } fx + v\varphi x$$

and $$fx \text{ becomes } fx + v\varphi x$$

then $\underset{1}{fax}$ will be $\underset{1}{fax} + v\underset{1}{\varphi ax}$

and $\underset{2}{fax}$ will be $\underset{2}{fax} + v\underset{2}{\varphi ax}$

and these being substituted we have

$$\frac{(\underset{1}{fx}-fx\,.\,\underset{2}{fax})-(\underset{1}{\varphi x}-fx\,.\,\underset{1}{\varphi ax}-\underset{2}{fax}\,.\,\varphi x)v-\varphi x\,.\,\underset{2}{\varphi ax}\,.\,v^2}{(1-fx\,.\,\underset{2}{fax})-(\underset{2}{fx}\,.\,\varphi ax+\underset{2}{fax}\,.\,\varphi x)v-\varphi x\,.\,\varphi ax\,.\,v^2}$$

now in consequence of the two equations given above, the first term in both numerator and denominator vanishes, and dividing the remainder by v and then making $v=0$, we have for the value of the expression when $\underset{1}{fx}-fx\,.\,\underset{1}{fax}=0$ and also $fx\,.\,fax=1$

$$\frac{\underset{1}{fx}-fx\,.\,\underset{1}{fax}}{1-fx\,.\,\underset{2}{fax}}=\frac{\underset{1}{\varphi x}-fx\,.\,\underset{1}{\varphi ax}-\underset{1}{fax}\,.\,\varphi x}{\underset{2}{fx}\,.\,\varphi ax+\underset{2}{fax}\,.\,\varphi x}$$

where φ and $\underset{1}{\varphi}$ are arbitrary.

If we take the expression

$$\frac{\underset{1}{fx}-fx\,.\,\underset{1}{fax}+fx\,.\,\underset{1}{fax}\,.\,\underset{2}{fa^2x}}{1+fx\,.\,\underset{1}{fax}\,.\,\underset{2}{fa^2x}}$$

and if fx and $\underset{1}{fx}$ are so assumed that the numerator and denominator both vanish, by a similar mode of treatment to that already pointed out, we shall find its real value to be

$$\frac{\underset{1}{\varphi x}-fx\,.\,\underset{1}{\varphi ax}-\underset{1}{fax}\,.\,\varphi x-fx\,.\,\underset{1}{fax}\,.\,\underset{1}{\varphi ax}-\underset{1}{fax}\,.\,\underset{1}{fax}\,.\,\varphi x-fx\,.\,\underset{1}{fax}\,.\,\underset{2}{\varphi a^2x}}{\underset{1}{fax}\,.\,\underset{2}{fa^2x}\,.\,\varphi x+fx\,.\,\underset{2}{fa^2x}\,.\,\varphi ax+fx\,.\,\underset{1}{fax}\,.\,\underset{2}{fa^2x}}$$

And similarly if we supposed both numerator and denominator of the fraction

$$\frac{\underset{1}{fx}-fx\,.\,\underset{1}{fax}+\&c.\pm fx\,.\,fax\ldots\underset{n-2}{fa^{n-2}x}\,.\,\underset{n-1}{fa^{n-1}x}}{1\pm fx\,.\,fax\ldots\underset{n-1}{fa^{n-1}x}}$$

to vanish, it would yet retain a value containing arbitrary functions.

It will be needless to multiply examples, as the mode of

treating them is sufficiently obvious from those already given. It appears then, that as in common algebra an expression may become illusory from the *variable quantity* assuming a particular *value*; so in the doctrine of functions an expression may become likewise illusory by the *variable function* assuming a particular *form*; in the one case the real value may be a *constant quantity*, in the other it may be an *arbitrary function*: nor ought this circumstance of the appearance of an arbitrary function to surprise us, when we consider that (as for instance in the second example) it is not one form only of the function fx which will satisfy the equation $fx \cdot fax = 1$, but any of the infinite variety of forms comprehended in the expression $fx = \left\{ \chi\left(\overline{x}, \overline{ax}\right) \right\}^{x-ax}$ and similarly for the values of fx. The circumstance of this species of vanishing fractions having an arbitrary function contained in their true value, is of considerable importance in the calculus of functions, as I shall now shortly endeavour to prove.

The Royal Society did me the honour to insert, in the last volume of their Transactions, a paper of mine, in which I gave a new method of solving all functional equations of the first order, and of a certain class by means of elimination, and I there stated that all solutions so obtained were only particular cases of the general solutions, and that they did not contain any arbitrary function.

Now it may be observed, that there are certain forms which may be assigned to the coefficients which render those solutions apparently infinite, yet that on farther consideration it appears, that the solution is in fact a vanishing fraction: in all such cases the process I have just pointed out will give the real solution which will contain an arbitrary function, so that it is in fact a general solution. Thus in the equation $\downarrow x \dashv fx$.

$\psi ax = {}_1fx$ whose solution is $\psi x = \frac{{}_1fx - {}_1f_0 \cdot {}_1fax}{1 - fx \cdot fax}$ if $fx \cdot fax = 1$ it apparently becomes infinite, but by subtracting the equation $fx \cdot \psi ax + fx \cdot fax \cdot \psi a^2 x = fx \cdot {}_1fax$ (which is deduced from the former by putting ax for x and multiplying by fx) we have since $a^2 x = x$ and also $fx \cdot fax = 1$, $0 = {}_1fx - fx \cdot {}_1fax$, and the solution becomes a vanishing fraction whose value is $\psi x = \frac{{}_1\varphi x - fx \cdot {}_1\varphi ax - {}_1fax \cdot \varphi x}{fx \cdot \varphi ax + fax \cdot \varphi x}$ let $ax = -x$ and $fx = {}_1fx = 1$ then the general solution of the equation $\psi x + \psi(-x) = 1$ is $\psi x = \frac{{}_1\varphi x - {}_1\varphi(-x) + \varphi x}{\varphi(-x) + \varphi x}$.

In a similar manner the solutions of the equations $\psi x - \psi(-x) = x$, and $\psi x = fx \cdot \psi ax$ if $fx \cdot fax = 1$ when $a^2 x = x$ are

$$\psi x = \frac{x\varphi x - {}_1\varphi x + {}_1\varphi(-x)}{\varphi x + \varphi(-x)} \text{ and } \psi x = \frac{{}_1\varphi x + fx \cdot {}_1\varphi ax}{fx \cdot \varphi ax + fax \cdot \varphi x}$$

Let $\psi x + \psi ax = {}_1fx$ if $a^2 x = x$ this is only possible when ${}_1fx = {}_1f(\overline{x, ax})$ then

$$\psi x = \frac{\varphi x \cdot {}_1f(\overline{x, ax})}{\varphi x + \varphi ax}$$

Let $\psi x - \psi ax = {}_1fx$ and $a^2 x = x$ this is only possible when ${}_1fx = (x - ax) \times {}_1f(\overline{x, ax})$, then

$$\psi x = \frac{(x - ax) \cdot {}_1f(\overline{x, ax}) \cdot {}_1\varphi x + \varphi x + {}_1\varphi ax}{\varphi x + \varphi ax}$$

The above are sufficient as examples, but the same reasoning has led me to the following curious proposition. *Whenever the method of elimination apparently fails, the real value of the vanishing fraction will give the general solution of the equation.* This principle puts us in possession of the general solutions of several classes of equations, for besides the cases

in which the solutions vanish from some particular values of the coefficients, *all equations which are homogeneous relative to the different forms of the unknown function* are comprehended in it, as are also *all equations which are symmetrical relative to the same quantities*.

There exists another class of equations nearly allied to those which are symmetrical relative to the different forms of the unknown functions, whose solution I shall now point out, chiefly with the view of giving another example of a method of reasoning which may frequently be employed with advantage in these inquiries, and also for the purpose of mentioning a remark respecting the elimination of variables in a certain class of algebraic equations which I do not recollect to have seen noticed. The class of functional equations to which I allude, are contained in the expression

$$\mathrm{F}\left\{\psi x, \psi \alpha x, \ldots \psi \alpha^{n-1} x, \overline{x}, \overline{\alpha x}, \ldots \overline{\alpha^{n-1} x}\right\} = 0$$

which for the sake of convenience may be written thus,

$$\underset{1}{\mathrm{F}}\left\{\psi x, \psi \alpha x, \ldots \psi \alpha^{n-1} x, fx, \underset{1}{fx}, \underset{2}{fx}, \&c.\right\} = 0$$

where $\alpha^n x = x$ and $fx, \underset{1}{fx}, \underset{2}{fx}$, &c. are any symmetrical functions of $x, \alpha x, \ldots \alpha^{n-1} x$.

In this equation none of the known functions $fx, \underset{1}{fx}$, &c. are changed by the substitution of $\alpha x, \alpha^2 x, \ldots \alpha^{n-1} x$ for x; and since the form of the unknown function depends on that of those which are known, it follows that the form of the unknown function will not be changed by the substitution of $\alpha x, \alpha^2 x, \ldots \alpha^{n-1} x$ for x, or in other words we may suppose $\psi x = \psi \alpha x = \&c. = \psi \alpha^{n-1} x$, and consequently ψx will be determined by the equation

$$\underset{1}{\mathrm{F}}\left\{\psi x, \psi x, \ldots \psi x, fx, \underset{1}{fx}, \underset{2}{fx}, \&c.\right\} = 0$$

or from $F\left\{\psi x, \psi\alpha x, \ldots \psi\alpha^{n}x, \overline{x}, \overline{\alpha x}, \overline{\alpha^{2}x}, \ldots \overline{\alpha^{n-1}x}\right\} = 0$.

If there should exist any doubt respecting the accuracy of this reasoning, it may be confirmed by arriving at the same conclusion in rather a different manner. If in the given equation we substitute successively $\alpha x, \alpha^{2}x, \ldots \alpha^{n-1}x$ for x, we shall have the following equations.

$$F\left\{\psi x, \quad \psi\alpha x, \ldots \psi\alpha^{n-1}x, \overline{x}, \overline{\alpha x}, \ldots \overline{\alpha^{n-1}x}\right\} = 0$$

$$F\left\{\psi\alpha x, \quad \psi\alpha^{2}x, \ldots \psi x, \quad \overline{x}, \overline{\alpha x}, \ldots \overline{\alpha^{n-1}x}\right\} = 0$$

&c. &c.

$$F\left\{\psi\alpha^{n-1}x, \psi x, \ldots \psi\alpha^{n-2}x, \overline{x}, \overline{\alpha x}, \ldots \overline{\alpha^{n-1}x}\right\} = 0$$

To eliminate $\psi\alpha x, \psi\alpha^{2}x$ &c. from these n equations would in most cases be excessively troublesome. It may however be observed, that $\psi\alpha x$ occurs in the second exactly in the same manner that ψx occurs in the first; also $\psi\alpha^{2}x$ is contained in the third in the same manner as ψx is contained in the first, and similarly with the rest. From this it follows, that though no one of the equations is symmetrical relative to $\psi x, \psi\alpha x \ldots \psi\alpha^{n-1}x$, yet when all the equations are considered together, they are symmetrical relative to $\psi x, \psi\alpha x, \ldots \psi\alpha^{n-1}x$; from this it follows that whether we find by elimination $\psi x, \psi\alpha x, \ldots$ or $\psi\alpha^{n-1}x$ the result will be the same, therefore $\psi x = \psi\alpha x =$ &c. $= \psi\alpha^{n-1}x$, and we may determine ψx from the equation

$$F\left\{\psi x, \psi x, \ldots \psi x, \overline{x}, \overline{\alpha x}, \ldots \overline{\alpha^{n-1}x}\right\} = 0 \qquad (a)$$

It does not follow from hence, that this equation contains all the values of ψx: on the contrary, if the elimination between the n equations above written were actually performed, it would be found that the equation (a) would enter into the final result as one of its factors.

If we apply this reasoning to algebraic equations containing several variables, as for instance to the two equations

$$F(x, y, a, b) = 0, \quad F(y, x, a, b) = 0$$

we shall find that one set of the value of x is always contained in the equation $F(x, x, a, b) = 0$.

As an example, let us take the two equations

$$y^3 + x = a \text{ and } x^3 + y = a$$

one set of the values of x are contained in the equation

$$x^3 + x - a = 0$$

and this enters as a factor in the result of elimination, which gives

$$x^9 - 3ax^3 + x + a^3 - a = (x^3 + x - a)(x^6 - x^4 + ax^3 - x^2 + ax - a^2 + 1) = 0$$

Another curious analogy exists between the calculus of functions and common algebra in the similarity of the relations of the roots of unity to the solutions of the functional equation $\psi^n x = x$.

In the equation $r^n = 1$ it is known that if r be one of the roots then any power of r will also be a root, and if n is a prime number and r any root except unity, then $r, r^2, r^3, \ldots r^{n-1}$ will be all the different roots. Similarly in the functional equation $\psi^n x = x$ if ax be one form which satisfies it, $a^m x$ will also fulfill it, and if n is a prime number then $ax, a^2x, \ldots a^{n-1}x$ will all be different forms which satisfy the equation. This may be readily shown as follows, if ax is a solution, then

$$a(ax) \ldots (n)\, x = a^n x = x$$

Suppose $m = 2$ then

$$a^2(a^2) \ldots (n)\, x = a^{2n} x = a^n (a^n x) = a^n x = x$$

consequently $a^2 x$ is also a solution of $\psi^n x = x$

again $\alpha^n \alpha^n \dots (n)x = \alpha^{n \times m} = \alpha^n \alpha^n \dots (m)x =$
$= \alpha^n \alpha^n \dots (m-1)x =$ &c. $= \alpha^n \alpha^n x = \alpha^n x = x$
consequently whenever m is a whole number $\alpha^n x$ is a solution of $\psi^n x = x$.

If we take the particular case of $n = 3$ we have $\psi^3 x = x$ and $\alpha x = \frac{1}{1-x}$ is one of its solutions, therefore other solutions will be $\alpha x = \frac{1}{1-x}$, $\alpha^2 x = \frac{x-1}{x}$, $\alpha^3 x = x$.

These expressions when generalised by the introduction of an arbitrary function, do not give solutions which are irreducible to each other, nor do they even then in my opinion contain all possible solutions; by introducing an arbitrary function they become

$$\varphi^{-1}\left(\frac{1}{1-\varphi x}\right) \text{ and } \varphi^{-1}\left(\frac{\varphi x - 1}{\varphi x}\right)$$

the latter of which gives $\frac{1}{1-x}$ if we make $\varphi x = \frac{1}{x}$.

If we consider the equation $\psi^6 x = x$ one of its values is $\alpha x = \frac{3x-3}{x}$, this gives for the others.

$$\alpha x = \frac{3x-3}{x}, \ \alpha^2 x = \frac{2x-3}{x-1}, \ \alpha^3 x = \frac{3x-6}{2x-3}$$

$$\alpha^4 x = \frac{x-3}{x-2}, \ \alpha^5 x = \frac{-3}{x-3}, \ \alpha^6 x = x$$

All these forms will satisfy the equation $\psi^6 x = x$ and they may all be generalised by the introduction of an arbitrary function similar to that employed for the equation $\psi^3 x = x$, but I do not suppose these solutions when thus generalised would contain all possible ones.

Perhaps the following observations may throw some light on the generality of the solutions of such equations as those we are now considering.

In the first place, it is obvious that every solution of $\psi^3 x = x$

will also be a solution of $\psi^{n}x = x$, such likewise will be all the solutions of $\psi^{r}x = x$. The complete solution of $\psi^{n}x = x$ should therefore contain all forms of x which satisfy the equations $\psi^{s}x = x$ and $\psi^{t}x = x$. Again, if a function as αx satisfies the equation $\psi^{2}x = -x$, it will also satisfy $\psi^{4}x = x$, for since $\alpha^{2}x = -x$ by putting $\alpha^{2}x$ for x it becomes $\alpha^{2}\alpha^{2}x = -\alpha^{2}x = -(-x) = x$, so also any function which satisfies the equation $\psi^{2}x = \frac{a}{x}$ will fulfil the equation $\psi^{4}x = x$ which may be proved in the same manner. I shall however take the more general case, and show that if any function satisfies the equation $\psi^{2}x = \beta x$ where β is any function satisfying the equation $\psi^{n}x = x$, it will also satisfy the equation $\psi^{n}x = x$, for since $\alpha^{2}x = \beta x$, putting $\alpha^{2}x$ for x, we have $\alpha^{2}\alpha^{2}x = \alpha^{2}x = \beta\alpha^{2}x = \beta\beta x = \beta^{2}x = x$. In the same way it might be demonstrated, that the complete solution of $\psi^{abc,\,\&c.}(x) = x$ must contain all functions which can satisfy any of the following conditions

$$\psi^{a}x = \alpha x \qquad \psi^{a'}x = \alpha' x \qquad \&c.$$

$$\psi^{b}x = \beta x \qquad \psi^{b'}x = \beta' x \qquad \&c.$$

$$\&c. \qquad \&c.$$

where α is any function satisfying the equation $\psi^{bc\,\&c.}(x) = x$

β	ditto	ditto	$\psi^{ac\,\&c.}(x) = x$
&c.	&c.		
α'	ditto	ditto	$\psi^{cd\,\&c.}(x) = x$
β'	ditto	ditto	$\psi^{ad\,\&c.}(x) = x$
&c.	&c.		&c.

The comparison of the integral calculus with that of functions will supply us with several very marked analogies, some of which promise when farther pursued to be of essen-

tial service in the improvement of this latter branch of analytical science. One of the first which presents itself is the method of solving the differential equation

$$o = ydx^n + A dx^{n-1}dy + B dx^{n-2}d^2y + \&c. + N d^n y + X dx^n$$

compared with that of solving the functional equation

$$o = \psi x + A\psi\alpha x + B\psi\alpha^2 x + \&c. + N\psi\alpha^{n-1}x + X$$

EULER and D'ALEMBERT succeeded in integrating the differential equation when all the coefficients except X are constant quantities, and LAGRANGE, in the Memoirs of the Academy of Berlin, 1772, explained a method of treating the same when they are variable; both the processes alluded to depend in the first instance on reducing the equation to the solution of the same equation, wanting the last term; and this is effected by means of a particular integral of the given equation.

The solution of the functional equation is precisely similar, it is first reduced to the solution of

$$o = \psi x + A\psi\alpha x + B\psi\alpha^2 x + \&c. + N\psi\alpha^{n-1}x$$

and this is effected by knowing a particular function which satisfies the original equation: this process I have already given in a former paper. The resemblance between the solutions of the two equations continues also in this respect, if fx is a particular case of the given equation, and Kx, Kx, Kx, &c. are particular cases of the same equation without its last term, then

$$\psi x = fx + Kx \cdot \chi(x, \alpha x, \ldots \alpha^{n-1}x) + Kx \cdot \chi(x, \alpha x, \ldots \alpha^{n-1}x) + \&c.$$

is a general solution of the equation.

It may be observed that the functional equation under consideration comprehends a large class of others which may easily be reduced to it, such as

$$F\psi x + A\,F\psi\alpha x + B\,F\psi\alpha^2 x + \&c. + N\,F\psi\alpha^{n-1}x + X = o$$

or more generally

$$F(x, \psi x) + A\,F(\alpha x, \psi\alpha x) + B\,F(\alpha^2 x, \psi\alpha^2 x) + \&c.$$
$$+ N F(\alpha^{n-1}x, \psi\alpha^{n-1}x) + X = o$$

both of which may be reduced to the same equations wanting the last term, these transformations apply with advantage to a great variety of other equations: the solution of the latter of these equations may be reduced to that of

$$\psi x + A\psi\alpha x + \&c. + N\psi\alpha^{n-1}x = o$$

and if we find $\psi x = Kx$ we shall have

$$\psi x = F^{1,-1}(x, Kx)$$

for the solution of the given equation.

As an instance of the utility of the former equation we may take $\left\{\psi(x)\right\}^2 + \left\{\psi\left(\frac{\pi}{2} - x\right)\right\}^2 = 1$

whose solution will be found by that method to be

$$\psi x = \sqrt{\frac{1}{2} + \left(\varrho x - \frac{\pi}{2}\right)\chi\left(x, \frac{\pi}{2} - x\right)}$$

the same equation or the more general one

$$\left\{\psi x\right\}^n + \left\{\psi\alpha x\right\}^n = 1$$

where $\alpha^2 x = x$ may be solved by another process which requires the aid of vanishing fractions, and we shall have

$$\psi x = \left\{\frac{\varrho x - \varrho\alpha x + \varrho x}{\varrho x + \varrho\alpha x}\right\}^{\frac{1}{n}}$$

The analogy between the various orders of functions which contain many variables, and that branch of the integral calculus which relates to *partial differentials*, is too apparent to require illustration. I shall therefore proceed to show that a functional equation admits of three species of solutions. 1st. *The complete solution, this contains as many arbitrary functions as the nature of the given equation will admit.* 2d. *The particular case, this contains all solutions which are less general than the complete solution, and which are only particular cases of it.* If they contain arbitrary functions, I have called them, for the sake of convenience, *general solutions*. 3d. *The particular solution, this is a solution which satisfies the equation, and may or may not contain arbitrary functions; its peculiar property is, that it is found from one part only of the equation and independent of the rest;* thus if we have the equation

$$\left\{\psi^{1,0}(x,y)-y\right\}\cdot \mathrm{F}\left\{x,y,\psi(x,y),\psi^{2,1}(x,y)\ \&c.\right\}$$
$$=\left\{\psi^{2,0}(x,y,)-\vec{\varphi}\,(o)\right\}\cdot \mathrm{F}\left\{x,y,\psi(x,y),\&c.)\right\}$$

$\psi(x,y)=\vec{\varphi}\,(\varphi x-\varphi y)$ is a particular solution, for it satisfies the equation by making $\psi^{1,0}(x,y)-y=o$ and $\overline{\psi^{2,0}}(x,y)-\vec{\varphi}^{1}(o)=o$, and is totally independent of the rest of the equation contained in the given function F and F, provided only that it does not make either of them infinite, whether it is contained in the complete solution, I have not yet ascertained; but I am rather of opinion, that it will be found not to be included in it. In both parts of my Essay towards the Calculus of Functions, I have used the expression, particular solution, instead of particular case, this arose from not having taken a sufficiently extensive view of the calculus; it would

perhaps be desirable to confine the meaning of *particular solution* to the definition which has just been given. It is needless to add that the different species of solutions just enumerated, bear a strong resemblance to those of differential equations.

Amongst differential equations containing more than two independent variables, a large proportion do not admit of any integrals, these can only be integrated by assigning some relation between the variables, an analogous case occurs with respect to functional equations, a large number of those which contain two or more variables admit of no solution, unless we assign some relation between the variable quantities; examples of such equations may be found in the second part of the Essay towards the Calculus of Functions. And here perhaps it may not be misplaced to state a difficulty of a peculiar nature with respect to functional equations which are impossible; for the sake of perspicuity I shall consider a very simple case

$$\psi x = c\psi\tfrac{1}{x}$$

for x substitute $\frac{1}{x}$ $\qquad \psi\frac{1}{x} = c\psi x$

and by multiplication $\psi x \times \psi\frac{1}{x} = c^2\psi\frac{1}{x} \times \psi x$ or $1 = c^2$ from which it follows that $c = \pm 1$ or in other words that the equation $\psi x = c\psi\frac{1}{x}$ is contradictory unless $c = \pm 1$. Now the functional equation $\mathrm{F}\left\{x, \psi x, \psi\alpha x\right\} = o$ has been reduced by Laplace by means of a very elegant artifice to an equation of finite differences, nor am I aware that this profound analyst has pointed out any restriction or any

impossible case: if we treat the equation $\psi x = c\psi x^n$ by his method, we shall find for its solution

$$\psi x = c^{\frac{-\log \log x}{\log n}}$$

and this solution satisfies the equation $\psi x = c\psi x^n$ independently of any particular value of n, and if we suppose $n = -1$ we have

$$\psi x = c^{\frac{-\log^2 x}{\log n}}$$

for the solution of the equation $\psi x = c\psi \frac{1}{x}$ whatever may be the value of c, and we have before shown that it cannot have a solution unless $c = \pm 1$. The only explanation I am at present able to offer concerning this contradiction, is one which I hinted at on a former occasion, viz. that if we suppose ψ to represent any inverse operation which admits of several values, then if throughout the whole equation we always take the same root or the same individual value of ψ, it is impossible to satisfy the equation, but if we take one value of ψ in one part, and another of the values of ψ in other parts of the equation, it is possible to fulfil it by such means. This solution may perhaps appear unsatisfactory, it is however only proposed as one which deserves examination, and I shall be happy if its insufficiency shall induce any other person to explain more clearly a very difficult subject.

One of the most extensive methods of integrating differential equations, consists in multiplying by such a factor as will render the whole equation a complete differential, the determination of this factor is, however, generally a matter of at least equal difficulty with that of solving the original equation: analogy would lead us to suspect that some similar mode might be adopted for the solution of functional equa-

tions, varying of course in a certain degree from the difference of the objects to be obtained: the theory which I am now to explain will show that this suspicion is not without foundation, and will at the same time unfold one of the most beautiful parallels between the integral and functional calculus which I have yet observed. It has been already shown, that when an equation is symmetrical relative to the different forms of the unknown function, as

$$\mathrm{F}\left\{x, \overline{\psi x}, \overline{\psi\alpha x}, \ldots \overline{\psi\alpha^{n-1}x}\right\} = 0$$

the method of solution by elimination apparently becomes illusory, but that the general solution of the equation may be deduced from the vanishing fraction to which this method then leads. If therefore we can make any functional equation symmetrical, relative to the different forms of the unknown functions, we can then obtain its general solution. Now this may be effected by multiplying the equation by some factor, and determining this factor, so that the result shall be symmetrical. The discovery of this factor (as in the case of a differential equation) requires the solution of a functional equation of several variables, but fortunately the class of equations to which they belong are not of very great difficulty.

To begin with a very simple case, let us consider the equation $\psi x + fx \,.\, \overline{\psi\alpha x} = fx$ where $\alpha^n x = x$

multiplying by φx we have

$$\varphi x \,.\, \psi x + \varphi x \,.\, fx \,.\, \psi\alpha x = \varphi x \,.\, fx$$

now the first side of this equation will be symmetrical relative to ψx and $\psi\alpha x$, if we make

$$\varphi x = fx \,.\, \varphi\alpha x$$

this equation can only be possible when $fx \cdot fax = 1$ or when

$$fx = \left\{\chi\left(\overline{x}, \overline{ax}\right)\right\}^{a-ax}$$

and supposing $fx \cdot fax = 1$, we have $\phi x = \frac{1}{\sqrt{fx}}$ consequently the equation becomes

$$\frac{\psi x}{\sqrt{fx}} + \sqrt{fx} \cdot \psi ax = \frac{\psi x}{\sqrt{fx}} + \frac{\psi ax}{\sqrt{fax}} = \frac{fx}{\sqrt{fx}}$$

and since the left side of this equation is symmetrical, the right side must also be so, consequently

$\frac{fx}{\sqrt{fx}} = f\left(\overline{x}, \overline{ax}\right)$ or the given equation is impossible, unless

$$fx = \sqrt{fx} \cdot f\left(\overline{x}, \overline{ax}\right)$$

and the solution of the equation will be found by taking the value of a vanishing fraction to be

$$\psi x = \frac{\sqrt{fax} \cdot f\left(\overline{x}, \overline{ax}\right) \cdot \varphi x - \varphi x + fx \cdot \varphi ax}{fx \cdot \varphi ax + fax \cdot \varphi x}$$

This method is applicable to all equations of the first degree, but I shall now point out a principle which extends to all equations of the first order, and which may probably be applied with some modification to those of higher orders.

Let us consider the equation

$$F\left\{x, ax, \psi x, \psi ax\right\} = o, \text{ where } a^n x = x$$

multiply this by $\Phi\left\{x, ax, \psi x, \psi ax\right\} = o$

it becomes

$$F\left\{x, ax, \psi x, \psi ax\right\} \times \Phi\left\{x, ax, \psi x, \psi ax\right\} = o$$

in order that this equation may be symmetrical, we must have

$$F\left\{x, ax, \psi ax\right\} \times \Phi\left\{x, ax, \psi x, \psi ax\right\}$$
$$= F\left\{ax, x, \psi ax, \psi x\right\} \times \Phi\left\{ax, x, \psi ax, \psi x\right\}$$

from which the form of Φ must be determined, one value is

$$\Phi(x, ax, \psi x, \psi ax) = F\{ax, x, \psi ax, \psi x\}$$

hence the equation

$$F\{x, ax, \psi x, \psi ax\} \times F\{ax, x, \psi ax, \psi x\} = 0$$

is symmetrical, and its general solution may therefore be found. It must however be observed, that all the solutions of this latter equation are not necessarily solutions of the former one, and it may be a matter of some difficulty to ascertain what solutions ought to be excluded. There is no part of this process which limits it to the particular case we have considered, and if the given equation were

$$F\{x, \psi x, \psi ax, \ldots \psi a^{n-1}x\} = 0$$

we might take the equation

$$\Phi\{F\{x, \psi x, \psi ax, \ldots \psi a^{n-1}x\}, \overline{F\{ax, \psi ax, \ldots \psi x\}}, \ldots$$
$$\ldots\ldots\ldots\ldots \overline{F\{a^{n-1}x, \psi a^{n-1}x, \ldots \psi a^{n-2}x\}}\} = 0$$

which is symmetrical relative to ψx, ψax, &c. $\psi a^{n-1}x$.

I shall not at present enter into any farther details respecting this mode of solving functional equations, as it forms the subject of some investigations on which I am now engaged, and which are as yet incomplete.

In the preceding pages I have endeavoured to point out some of the more prominent points in which the calculus of functions resembles common algebra or the integral calculus: it ought however to be observed, that several of the methods which I have applied to the solution of functional equations, directly resulted from pursuing this analogy: for when I had ascertained the remarkable similitude which exists between

the method of functions and the integral calculus, I referred to a treatise on that subject with the express purpose of endeavouring to transfer the methods and artifices employed in the latter calculus, to the cultivation and improvement of the former.

CHARLES BABBAGE.

March 5, 1817.

XV. *Of the construction of Logarithmic Tables.* *By* Thomas Knight, *Esq.* *Communicated by* Taylor Combe, *Esq.* *Sec. R. S.*

Read February 27, 1817.

1. I HAVE endeavoured, in this short Paper, to give a *simple* and *connected* theory of the construction of logarithms, which I think has not hitherto been done.

PROP. I.

To find the Logarithm of $1+x$.*

It is not difficult to see that we may assume

$$\mathrm{L}(1+x) = {}'\mathrm{A}x + {}''\mathrm{A}x^2 + {}'''\mathrm{A}x^3 + {}''''\mathrm{A}x^4 + \&c.,\text{ whence}$$

$$\mathrm{L}(1+y) = {}'\mathrm{A}y + {}''\mathrm{A}y^2 + {}'''\mathrm{A}y^3 + {}''''\mathrm{A}y^4 + \&c.,\text{ and}$$

$\mathrm{L}\{(1+x)(1+y)\} = \mathrm{L}(1+x+y+xy)$, or putting $1+x=\pi$,

$$=\mathrm{L}\{1+(x+\pi y)\} = {}'\mathrm{A}(x+\pi y) + {}''\mathrm{A}(x+\pi y)^2 + {}'''\mathrm{A}(x+\pi y)^3 + \&c.$$

If we substitute these three expansions in the equation

$$\mathrm{L}(1+x) + \mathrm{L}(1+y) = \mathrm{L}\{(1+x)(1+y)\}$$

which expresses the nature of logarithms, and compare the coefficients of the first power of y, we find

$${}'\mathrm{A} = {}'\mathrm{A}\pi + 2{}''\mathrm{A}\pi x + 3{}'''\mathrm{A}\pi x^2 + 4{}''''\mathrm{A}\pi x^3 + \&c.$$

or
$$\begin{aligned} {}'\mathrm{A} = {}'\mathrm{A} &+ 2{}''\mathrm{A}x + 3{}'''\mathrm{A}x^2 + 4{}''''\mathrm{A}x^3 + \&c. \\ &+ {}'\mathrm{A}\,| \quad + 2{}''\mathrm{A}\,| \quad + 3{}'''\mathrm{A}\,| \end{aligned}$$

whence, by comparing the coefficients of the powers of x,

${}'\mathrm{A} = {}'\mathrm{A}$, $2{}''\mathrm{A} + {}'\mathrm{A} = 0$, $3{}'''\mathrm{A} + 2{}''\mathrm{A} = 0$, $4{}''''\mathrm{A} + 3{}'''\mathrm{A} = 0$, &c.; or

${}'\mathrm{A} = {}'\mathrm{A}$, ${}''\mathrm{A} = -\frac{{}'\mathrm{A}}{2}$, ${}'''\mathrm{A} = -\frac{2{}''\mathrm{A}}{3} = \frac{{}'\mathrm{A}}{3}$, ${}''''\mathrm{A} = -\frac{3\mathrm{A}'''}{4} = -\frac{{}'\mathrm{A}}{4}$, &c.

* I find that the method of expansion made use of in this Proposition had been previously employed by Mr. STRINGS.

and $\mathrm{L}(1+x) = {}'\mathrm{A}\left\{\frac{x}{1} - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \right\}$

As for 'A it may be evidently taken at pleasure; and innumerable systems of logarithms may be formed by assigning different values to it, for

$${}'\mathrm{A}\,\mathrm{L}(1+x) + {}'\mathrm{A}\mathrm{L}(1+y) = {}'\mathrm{A}\mathrm{L}\left\{(1+x)(1+y)\right\}$$

which expresses that, if every logarithm in a system be multiplied by the same constant quantity, the products will still form a system of logarithms to the same numbers.

Cor. By an easy transformation of $\mathrm{L}(1+x)$, we get for BRIGG's logarithms, M being the modulus,

$\mathrm{L}.\frac{a}{b} = 2\,\mathrm{M}\left\{\frac{a-b}{a+b} + \frac{1}{3}\left(\frac{a-b}{a+b}\right)^3 + \frac{1}{5}\left(\frac{a-b}{a+b}\right)^5 + \right\}$; and whenever the logarithm of a fraction is spoken of in the following proposition, it is supposed to be found by this series.

2. How are we to begin, in forming a table of logarithms?

DELAMBRE (Preface to BORDA, p. 75) says, that we should begin at 10,000; and the same writer (Mémoires de l'Institut, Tome cinq. p. 65), speaking of the great French Tables, says that the logarithms of primes under 10,000 were calculated directly by series, and those of numbers above 10,000 by six orders of differences.

Now it is not easy to see, why any of the logarithms in the lower half of the Table, except those of the numbers 2 and 3, should be computed directly; since they may be got, each by a single subtraction, from those in the upper half. Suppose, for instance, there had been found directly the logarithms of numbers from 100,000 to 200,000; those of numbers down to 50,000 are found by merely subtracting the logarithm of 2, successively, from those of all the even numbers;

beginning at the top of the Table, with L. 1999998, L. 1999996, &c., and setting down the remainder for the logarithms of the successive numbers below 100,000, viz. L. 99999, L. 99998, &c.

When we have got down to 50,000, if we were to proceed in the same way, we should have to operate on the logarithms thus obtained, between 100,000 and 50,000: If, therefore, we fear any accumulation of errors, we may (because $3 \times 49999 = 149997$) subtract the logarithm of 3 from L. 149997, and from the logarithm of every third number going downward, and set the remainders down successively for the logarithms of numbers below 50,000. And thus we may proceed till we get somewhat below 34,000; then the logarithm of 4 will carry us down to 25,000; and the logarithm of 5 to 20,000, which completes the work, those below 20,000 having been already found.

In the great French Tables, however, it has been thought proper to calculate the logarithms of numbers under 10,000 with more decimal places than the rest. These must necessarily be found independently of the others; as they form in reality a separate Table.

In the next proposition, is contained a general method of finding converging series for the calculation of logarithms. The propositions which follow this are only corollaries from it, and give forms for interpolation; so that *every thing* relating to the construction of logarithms is effected by one simple and uniform process.

PROP. II.

8. *To express a number* (x) *by the product of a series of fractions converging continually towards unity.*

Let n, n', n'', &c. be numbers much less than x; in the equation $x=x$, change x, in the second member, into $x+n$, and multiply by such a factor as will restore the equality; there arises $x=(x+n)\times\frac{x}{x+n}$. If, in the second member of this equation, we change x into $x+n'$, in the *last factor* $\frac{x}{x+n}$, and multiply by such a fractional factor as shall again restore the equality, we have

$$x=(x+n)\times\frac{x+n'}{x+n+n'}\times\frac{x(x+n+n')}{(x+n)(x+n')}.$$

If here, in like manner, we change x into $x+n''$ in the last factor, and restore the equality as before, by annexing a new factor, then

$$x=(x+n)\times\frac{x+n'}{x+n+n'}\times\frac{(x+n'')(x+n+n'+n'')}{(x+n+n'')(x+n'+n'')}\times\frac{x(x+n+n')(x+n+n'')(x+n'+n'')}{(x+n)(x+n')(x+n'')(x+n+n'+n'')}$$

and the same process may be repeated as long as we think it necessary. Now it is plain that the last annexed factor, as we continue these operations, must always approach nearer to unity than that which was the last before; thus, n being very small compared with x, $\frac{x}{x+n}$ does not much differ from unity, and when $x+n'$ is put for x in this fraction (n' being also very small compared with x) its value will be nearly the same as before: of course the annexed factor $\frac{x(x+n+n')}{(x+n)(x+n')}$ will differ very little from unity: and it will differ from it

much less than the preceding factor $\frac{x}{x+n}$; for let $\frac{x}{x+n}=1-\mu$, $\frac{x+n'}{x+n+n'}=1-\mu'$, and μ and μ' being small fractions; the new factor $\frac{x(x+n+n')}{(x+n)(x+n')}=\frac{1-\mu}{1-\mu'}=1-(\mu-\mu')$ nearly: and consequently differs less from unity than the factor which was last before.

4. These equations, being put into logarithms, give a series of converging expressions for the logarithm of x. We have successively,

1st. $\mathrm{L}.x=\mathrm{L}(x+n)+\mathrm{L}\left(\frac{x}{x+n}\right)$

2d. $\mathrm{L}.x=\mathrm{L}(x+n)+\mathrm{L}(x+n')-\mathrm{L}(x+n+n')+\mathrm{L}\left(\frac{x^2+(n+n')x}{x^2+(n+n')x+nn'}\right)$;

but before we put the third equation into logarithms, it will be better to simplify it; one of the most obvious ways of doing which is to make $n+n'=n''$; then

3d. $\mathrm{L}.x=\mathrm{L}(x+n)+\mathrm{L}(x+n')+\mathrm{L}(x+2n'')-\mathrm{L}(x+n+n'')-\mathrm{L}(x+n'+n'')+\mathrm{L}\left(\frac{x^3+3n''x^2+(nn'+2n''^2)x}{x^3+3n''x^2+(nn'+2n''^2)x}\right)$.

This may be still farther simplified by making $n=n'$, consequently $n''=2n$, then

$$\mathrm{L}.x=2\mathrm{L}.(x+n)-2\mathrm{L}(x+3n)+\mathrm{L}(x+4n)+\mathrm{L}\left(\frac{x^3+6nx^2+9n^2x}{x^3+6nx^2+9n^2x+4n^3}\right)$$

If now we change n into -1, and x into $x+2$, we shall fall upon the elegant formula of Mr. Borda.*

* If any one shall attempt to calculate a Table of logarithms by means of differences, each logarithm being got, not from the next, but from the next but one below it, he will fall upon the series of Borda: for $\Delta^1.\mathrm{L}(x-1)=\mathrm{L}.(x+2)-3\mathrm{L}(x+1)+3\mathrm{L}.x-\mathrm{L}(x-1)$; $\Delta^2.\mathrm{L}(x-2)=\mathrm{L}(x+1)-3\mathrm{L}.x+3\mathrm{L}(x-1)-\mathrm{L}(x-2)$, by adding which, $\Delta^1.\mathrm{L}(x-1)+\Delta^2.\mathrm{L}(x-2)=\mathrm{L}\left(\frac{(x+2)(x-1)^3}{(x-2)(x+1)^3}\right)$ which gives the series we are speaking of. This remark will be exemplified in one of the following Propositions.

In like manner we might investigate approximations of the fourth and following orders: but this kind of research has very little use, and the Proposition was inserted for a different purpose.

PROP. III.

5. *Supposing that, in the last Proposition,* $n=n'=n''=\&c.=-1$. *It is required to find the law of the converging expressions for* L.x.

In this case the four first transformations give

$$x=(x-1)\times\frac{x}{x-1}$$

$$x=(x-1)\times\frac{x-1}{x-2}\times\frac{x(x-2)}{(x-1)^2}$$

$$x=(x-1)\times\frac{x-1}{x-2}\times\frac{(x-1)(x-3)}{(x-2)^2}\times\frac{x(x-2)^2}{(x-1)^2(x-3)}$$

$$x=(x-1)\times\frac{x-1}{x-2}\times\frac{(x-1)(x-3)}{(x-2)^2}\times\frac{(x-1)(x-3)^2}{(x-2)^2(x-4)}\times\frac{x(x-2)^3(x-4)}{(x-1)^3(x-3)^2}$$

which, put into logarithms, give

$$\mathrm{L}.x=\mathrm{L}(x-1)+\mathrm{L}\left(\frac{x}{x-1}\right)$$

$$\mathrm{L}.x=2\mathrm{L}(x-1)-\mathrm{L}(x-2)+\mathrm{L}\left(\frac{x(x-2)}{(x-1)^2}\right)$$

$$\mathrm{L}.x=3\mathrm{L}(x-1)-3\mathrm{L}(x-2)+\mathrm{L}(x-3)+\mathrm{L}\left(\frac{x(x-2)^3}{(x-1)^2(x-3)}\right)$$

$$\mathrm{L}.x=4\mathrm{L}(x-1)-6\mathrm{L}(x-2)+4\mathrm{L}(x-3)-\mathrm{L}(x-4)+\mathrm{L}\left(\frac{x(x-2)^4(x-4)}{(x-1)^4(x-3)^3}\right);$$

where a coincidence may be observed between the coefficients and those in the binomial theorem; and it is easily shown that the same coincidence will have place, how far soever we continue the method; or that, in general, the converging expression will be

$$\mathrm{L}.x=\frac{n}{1}\mathrm{L}(x-1)-\frac{n(n-1)}{1.2}\mathrm{L}(x-2)+\frac{n(n-1)(n-2)}{1.2.3}\mathrm{L}(x-3)-\ldots$$

$$\ldots\ldots+\mathrm{L}\left\{\frac{x\times(x-2)^{\frac{n(n-1)}{1.2}}\times(x-4)^{\frac{n(n-1)(n-2)(n-3)}{1.2.3.4}}+\&c.}{(x-1)^{\frac{n}{1}}\times(x-3)^{\frac{n(n-1)(n-2)}{1.2.3}}\times\&c.}\right\}(a)$$

For, if it be denied, let this represent a single result, to pass on to the next, we change x into $x-1$, in the logarithm of the fraction, and add a new logarithm (L) to restore the equality: the equation will thus become

$$\text{L}.x=\left.\begin{matrix}\frac{n}{1}\\ +1\end{matrix}\right\}\text{L}(x-1)\left.\begin{matrix}-\frac{n(n-1)}{1.2}\\ -\frac{n}{1}\end{matrix}\right\}\text{L}(x-2)\left.\begin{matrix}+\frac{n(n-1)(n-2)}{1.2.3}\\ +\frac{n(n-1)}{1.2}\end{matrix}\right\}\text{L}(x-3)-\ldots+(\text{L})$$

$$\text{or L}.x=\tfrac{n+1}{1}\text{L}(x-1)-\tfrac{(n+1)n}{1.2}\text{L}(x-2)+\tfrac{(n+1)n(n-1)}{1.2.3}\text{L}(x-3)-\ldots+(\text{L}),$$

and, by transposition, we find

$$(\text{L})=\text{L}\left\{\begin{matrix}\frac{x\times(x-2)}{1}\times\frac{(n+1)n}{1.2}\times(x-4)\times\frac{(n+1)n(n-1)(n-2)}{1.2.3.4}\times\text{&c.}\\ \hline \frac{(x-1)^{n+1}}{1}\times(x-3)\times\frac{(n+1)n(n-1)}{1.2.3}\times\text{&c.}\end{matrix}\right\}$$

so that the whole expression is of the same form as before, which is therefore proved to be general.

6. Cor. 1. If in the values of x. in the last article, we put for x, in the second $x+1$, in the third $x+2$, in the fourth $x+3$, and so on; and moreover represent the last fractions arising after such substitution by a, a', a'', a''', &c., we get the following set of equations

$$\left\{\begin{matrix}x=(x-1)\times a\\ x+1=x\times a\times a'\\ x+2=(x+1)\times\frac{x+1}{x}\times a'\times a''\\ x+3=(x+2)\times\frac{x+2}{x+1}\times\frac{x(x+2)}{(x+1)^2}\times a''\times a'''\\ \text{&c.}\quad\ldots\ldots\ldots\quad\text{&c.}\quad\ldots\ldots\end{matrix}\right.\qquad(b)$$

which, *from the manner of their formation*, are subject to this law, that the m^{th} fraction (provided it is not the last) in the value of $x+n$, is equal to $x+n-1$ divided by the product of the first $m-1$ fractions in the expression of $x+n-1$.

If, for brevity, we put L, L°, L′, L″, &c. for $L(x-1)$, $L.x$, $L(x+1)$, $L(x+2)$, &c. the last equations give

$$L^\circ = L + L.\alpha$$

$$L' = L^\circ + L.\alpha + L\alpha' \qquad (c)$$

$$L'' = L' + (L' - L^\circ) + L.\alpha' + L.\alpha''$$

$$L''' = L'' + (L'' - L') + (L'' - 2L' + L^\circ) + L.\alpha'' + L.\alpha'''$$

$$L'''' = L''' + (L''' - L'') + (L'' - 2L'' + L') + (L''' - 3L'' + 3L' - L^\circ) + L.\alpha''' + L.\alpha''''$$

. . . . . .

$$L^{\cdots(n+1)} = L^{\cdots n} + (L^{\cdots n} - L^{\cdots(n-1)}) + (L^{\cdots n} - 2L^{\cdots(n-1)} + L^{\prime\cdots(n-2)}) + \ldots\ldots + (L^{\cdots n} - nL^{\cdots(n-1)} + \frac{n(n-1)}{1.2}L^{\cdots(n-2)} - \frac{n(n-1)(n-2)}{1.2.3}L^{\cdots(n-3)} +) + L.\alpha^{\cdots n} + L.\alpha^{\cdots(n+1)}$$

These equations are subject to a law arising from that which we noticed in (*b*), viz. that the m^{th} term (provided it is not the last) in the value of $L^{\cdots n}$ is equal to $L^{\cdots(n-1)}$, minus the sum of the first $m-1$ terms in the expression $L^{\cdots(n-1)}$. By *term* I here mean the whole expression included between two brackets.

If we form $L\alpha^{\cdots n}$ generally from the last term of equation (*a*) we have

$$L.\alpha^{\cdots n} = L\left\{\frac{(x+1)\times(x+n-2)}{n+1} \times \frac{\frac{(n+1)n}{1.2}}{(x+n-1)} \times \frac{\frac{(n+1)n(n-1)(n-2)}{1.2.3.4}}{(x+n-3)} \times \&c. \right\}$$

$$+ \frac{\frac{(n+1)n(n-1)}{1.2.3}}{(x+n-3)} \times \&c.\Big\}$$

If any one should not be satisfied that the form given to $L^{\cdots(n+1)}$ is general, he has only to form $L^{\prime\cdots(n+2)}$ from it, and he will find the same form in that case. Now $L^{\cdots(n+2)}$ is formed from $L^{\cdots(n+1)}$ by changing x into $x+1$ (or $L^{\cdots r}$ into $L^{\cdots(r+1)}$ in all the terms but the last $L.\alpha^{\cdots(n+1)}$, and

this we may remember had x first changed into $x-1$, and afterwards into $x+1$, so that it receives no alteration. Finally there is to be added $\mathrm{L}.\alpha''\ldots(n+2)$.

7. *Cor.* 2. Change x into $x-1$ in equations (b), and let the new values of $\alpha, \alpha', \alpha''$, &c. be β, β', β'', &c., and there arises the following series of equations,

$$\begin{aligned}
x-1&=(x-2)\times\beta\\
x&=(x-1)\times\beta\times\beta'\\
x+1&=x\times\frac{x}{x-1}\times\beta'\times\beta''\\
x+2&=(x+1)\times\frac{x+1}{x}\times\frac{(x-1)(x+1)}{x^2}\times\beta''\times\beta'''\\
&\text{\&c.}\qquad\qquad\text{\&c.}
\end{aligned}$$

which being multiplied by equations (b), the first by the first, the second by the second, and so on, factor by factor, and putting $\alpha\times\beta=s$, $\alpha'\times\beta'=s'$, $\alpha''\times\beta''=s''$, and so on, give

$$\begin{aligned}
x&=(x-2)\times s\\
x+1&=(x-1)\times s\times s'\\
x+2&=x\times\frac{x+1}{x-1}\times s'\times s''\\
x+3&=(x+1)\times\frac{x+2}{x}\times\frac{(x+2)(x-1)}{(x+1)x}\times s''\times s'''\\
&\text{\&c.}\qquad\qquad\text{\&c.}
\end{aligned}$$

which, being put into logarithms, give

$$\begin{aligned}
\mathrm{L}^\circ&=\mathrm{L}+\mathrm{L}.s\\
\mathrm{L}'&=\mathrm{L}+\mathrm{L}.s+\mathrm{L}.s'\qquad(d)\\
\mathrm{L}''&=\mathrm{L}^\circ+(\mathrm{L}'-\mathrm{L})+\mathrm{L}.s'+\mathrm{L}.s''\\
\mathrm{L}'''&=\mathrm{L}'+(\mathrm{L}''-\mathrm{L}^\circ)+(\mathrm{L}''-\mathrm{L}'-\mathrm{L}^\circ+\mathrm{L})+\mathrm{L}.s''+\mathrm{L}.s'''\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
\mathrm{L}^{n\ldots(n+1)}&=\mathrm{L}^{n\ldots(n-1)}+(\mathrm{L}^{n\ldots n}-\mathrm{L}^{n\ldots(n-2)})+(\mathrm{L}^{n\ldots n}-\mathrm{L}^{n\ldots(n-1)}
\end{aligned}$$

$$-L''^{\cdots(n-2)}+L''^{\cdots(n-3)})+\ldots\ldots\ldots+(L''^{\cdots n}-(n-1)L''^{\cdots(n-1)}+$$
$$\frac{n\times(n-3)}{1.2}L''^{\cdots(n-2)}-\frac{n(n-1)\times(n-5)}{1.2.3}L''^{\cdots(n-3)}+)+L.s''^{\cdots n}+$$
$$+L.s''^{\cdots(n+1)};$$ any immediate term, as the $m+1^{th}$, will be of the form $L''^{\cdots n}-(m-1)L''^{\cdots(n-1)}+\frac{m\times(m-3)}{1.2}L''^{(n-2)}-$
$$\frac{m(m-1)\times(m-5)}{1.2.3}+.$$*

We easily see that

$$L.s''^{\cdots n}=L\left\{\frac{(n+1)\times(n-2)}{(x+n)\times(x+n-2)}\times\frac{(n+1)n(n-1)\times(n-6)}{(x+n-4)}\times\frac{}{1.2.3.4}\times\text{&c.}\right\}$$
$$\frac{(n+1)n\times(n-4)}{(x+n-1)^n\times(x+n-3)}\times\frac{}{1.2.3}\times\text{&c.}$$

It is scarcely necessary to say that $L.\alpha''^{\cdots n}$, $L.s''^{\cdots n}$ are to be expanded by the common series for $L.\frac{a}{b}$, viz.

* These forms are analogous to an expression in the method of differences, which, though not noticed by STIRLING and other writers on interpolation, may be useful on many occasions, as the coefficients are small and few in number. BORDA's expression for logarithms is a particular case of it.

$u_n=(n-1)u_{n-1}-\frac{n\times(n-3)}{1.2}u_{n-2}+\frac{n(n-1)\times(n-5)}{1.2.3}u_{n-3}-\frac{n(n-1)(n-2)\times(n-7)}{1.2.3.4}u_{n-4}$
$+\ldots\ldots\ldots+\Delta^n u+\Delta^n u_{-1}$. If we make $u_n=L.x$, we have, by taking n (in the coefficients) successively 1, 2, 3, 4, &c.

$L.x=L.(x-2)+$ series,

$L.x=L.(x-1)+L(x-2)-L(x-3)+$ series,

$L.x=2\left\{L(x-1)-L(x-3)\right\}+L(x-4)+$ series, (BORDA's if we change x into $x+2$).

$L.x=3\left\{L(x-1)+L(x-4)\right\}-2\left\{L(x-2)+L(x-3)\right\}-L(x-5)+$series,

$L.x=4\left\{L(x-1)-L(x-5)\right\}-5\left\{L(x-2)-L(x-4)\right\}+L(x-6)+$series,

$L.x=5\left\{L(x-1)+L(x-3)+L(x-4)+L(x-6)\right\}-9\left\{L(x-2)+L(x-5\right\}-L(x-7)+$series,

$L.x=6\left\{L(x-1)-L(x-7)\right\}-14\left\{L(x-2-L(x-3)+L(x-5)-L(x-6)\right\}+L(x-8)+$series,

&c. &c.

which may be useful, when taken without the series, as formulas of verification.

$\mathrm{L}.\frac{a}{b}=2\mathrm{M}\left\{\frac{a-b}{a+b}+\frac{1}{3}\left(\frac{a-b}{a+b}\right)^3+\frac{1}{5}\left(\frac{a-b}{a+b}\right)^5+\&c.\right\}$ though, for the purpose of the two following propositions, they will be better when expanded into series of monomials; see Prop. VI.

Prop. IV.

8. *To construct a Table of Logarithms by means of interpolation from the converging expressions* $\mathrm{L}.a$, $\mathrm{L}.a'$, $\mathrm{L}.a''$, &c.

When treating of the equations marked (c), we noticed a law to which the terms are subject; this law affords an easy method of eliminating the second, third, &c. terms, and, by this means, we find, successively,

$$\begin{aligned}
\mathrm{L}.x &= \mathrm{L}(x-1)+\mathrm{L}.a \\
\mathrm{L}(x+1) &= \mathrm{L}.x + \mathrm{L}.a + \mathrm{L}.a' \qquad (e)\\
\mathrm{L}(x+2) &= \mathrm{L}(x+1)+\mathrm{L}.a+2\mathrm{L}.a'+\mathrm{L}.a'' \\
\mathrm{L}(x+3) &= \mathrm{L}(x+2)+\mathrm{L}.a+3\mathrm{L}.a'+3\mathrm{L}.a''+\mathrm{L}.a''' \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
\mathrm{L}(x+n) &= \mathrm{L}(x+n-1)+\mathrm{L}.a+n\mathrm{L}.a'+\tfrac{n(n-1)}{1.2}\mathrm{L}.a''+
\end{aligned}$$

If any one doubts whether this form is general, for every value of n, let it be only a single value; and supposing a, a', a'', &c. to become $\underset{1}{a}$, $\underset{1}{a'}$, $\underset{1}{a''}$, &c., by the substitution of $x+1$ for x, we have

$$\mathrm{L}(x+n+1)=\mathrm{L}(x+n)+\mathrm{L}.\underset{1}{a}+n\mathrm{L}.\underset{1}{a'}+\tfrac{n(n-1)}{1.2}\mathrm{L}.\underset{1}{a''}+$$

Now, if we consider the manner in which the last fractional factors, in the values of x, at the beginning of the last proposition, were formed from one another; and the change which they afterwards underwent in forming equations (b), we shall easily perceive that

$\alpha''^{\cdots n} = \frac{\alpha''^{\cdots(n-1)}}{\alpha''^{\cdots(n-1)}}$; whence $L.\alpha''^{\cdots(n-1)} = L.\alpha''^{\cdots n} + L.\alpha''^{(n-1)}$

by means of which equation, $L(x+n+1)$ becomes

$$L(x+n+1) = L(x+n) + L.\alpha + \tfrac{n}{1} L.\alpha' + \tfrac{n(n-1)}{1.2} L.\alpha'' +$$
$$\tfrac{n(n-1)(n-2)}{1.2.3} L\alpha''' +$$
$$+ L.\alpha' + \tfrac{n}{1} L\alpha'' + \tfrac{n(n-1)}{1.2} L.\alpha''' +$$

or $L(x+n+1) = L(x+n) + L.\alpha + \frac{n+1}{1} L.\alpha' + \frac{(n+1)n}{1.2} L.\alpha'' +$ $\frac{(n+1)n(n-1)}{1.2.3} L.\alpha'''$ + the same form as before, which is therefore general.

9. By means of equations (e) the logarithm of a number is found from the logarithm of that which precedes it, and the process will consist of a number of additions, equal to the number of the values of α we make use of.

Thus, if $L.\alpha'''$ be so small, as not to affect the last figure, to which we intend to carry the logarithms, it may be neglected and we must make

$$L.\alpha + L.\alpha' = r$$
$$L.\alpha + 2L.\alpha' + L\,\alpha'' = r + L.\alpha' + L.\alpha'' = r'$$
$$L.\alpha + 3L.\alpha' + 3L.\alpha'' = r' + L\alpha' + 2L.\alpha'' = r''$$
$$L.\alpha + 4L.\alpha' + 6L.\alpha'' = r'' + L.\alpha' + 3L.\alpha'' = r'''$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$L\alpha + nL.\alpha' + \tfrac{n(n-1)}{1.2} L.\alpha'' = r''^{\cdots(n-2)} + L.\alpha' + (n-1)L.\alpha'' = r''^{\cdots(n-1)}$$

Here the quantities r', r'', r''', &c. are formed by two additions each, one more gives the logarithms; for, by substituting in (e),

$$L.x = L(x-1) + L.\alpha$$
$$L(x+1) = L.x + L\alpha + L\alpha' = L.x + r$$

$$L(x+2) = L(x+1) + r + L.\alpha' + L.\alpha'' = L(x+1) + r'$$
$$L(x+3) = L(x+2) + r' + L.\alpha' + 2L.\alpha'' = L(x+2) + r''$$
$$L(x+4) = L(x+3) + r'' + L.\alpha' + 3L.\alpha'' = L(x+3) + r'''$$

&c. &c. &c.

If, in the value of $L.\alpha''^{\ldots n}$ (Art. 6), we put successively 0, 1, 2 for n, we have

$$L.\alpha = L\left(\frac{x}{x-1}\right);\ L.\alpha' = L\left(\frac{(x-1)(x+1)}{x^2}\right);\ L.\alpha'' = L\left(\frac{(x+2)x^2}{(x-1)(x+1)^2}\right);$$

or

$$L.\alpha = 2M\left\{\frac{1}{2x-1} + \frac{1}{3}\left(\frac{1}{2x-1}\right)^3 + \frac{1}{5}\left(\frac{1}{2x-1}\right)^5 + \right\}$$
$$L.\alpha' = -2M\left\{\frac{1}{2x^2-1} + \frac{1}{3}\left(\frac{1}{2x^2-1}\right)^3 + \frac{1}{5}\left(\frac{1}{2x^2-1}\right)^5 + \right\}$$
$$L.\alpha'' = 2M\left\{\frac{2x+1}{2x^3+4x^2-2x-1} + \frac{1}{3}\left(\frac{2x+1}{2x^3+4x^2-2x-1}\right)^3 + \right\}$$

These are the most converging values, I shall show presently how to expand them into series of monomials.

10. If the intended number of decimal places should require $L.\alpha'''$ also to be retained, make, first

$$L.\alpha + L.\alpha' = r$$
$$L.\alpha + 2L.\alpha' + L.\alpha'' = r + L.\alpha' + L.\alpha'' = r'$$
$$L.\alpha + 3L.\alpha' + 3L.\alpha'' + L.\alpha''' = r' + L.\alpha' + 2L.\alpha'' + L.\alpha''' = r''$$
$$L.\alpha + 4L.\alpha' + 6L.\alpha'' + 4L.\alpha''' = r'' + L.\alpha' + 3L.\alpha'' + 3L.\alpha''' = r'''$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$L.\alpha + nL.\alpha' + \frac{n(n-1)}{1.2}L.\alpha'' + \frac{n(n-1)(n-2)}{1.2.3}L.\alpha''' = r''^{\ldots(n-2)} +$$
$$L\alpha' + (n-1)L.\alpha'' + \frac{(n-1)(n-2)}{1.2}L.\alpha''' = r''^{\ldots(n-1)}$$

Next make

$$L.\alpha' + L.\alpha'' = r^{(1)}$$
$$L.\alpha' + 2L.\alpha'' + L.\alpha''' = r^{(1)} + L.\alpha'' + L.\alpha''' = r'^{(2)}$$
$$L.\alpha' + 3L.\alpha'' + 3L.\alpha''' = r'^{(1)} + L.\alpha'' + 2L.\alpha''' = r''^{(2)}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\mathrm{L}.a'+(n-1)\mathrm{L}.a''+\frac{(n-1)(n-2)}{1.2}\mathrm{L}.a'''=r\,{}^{(1)}{}''\ldots\ldots(n-3)+\mathrm{L}.a''+$$

$$(n-2)\mathrm{L}.a''' \quad =r\,{}^{(2)}{}''\ldots\ldots(n-2)$$

If $\mathrm{L}.a''''$ had been used, we must have made $\mathrm{L}.a''+\mathrm{L}.a'''=r\,{}^{(3)}{}''$, and have proceeded as before.

The substitutions above being made equations (e) become

$$\mathrm{L}.x=\mathrm{L}(x-1)+\mathrm{L}.a$$

$$(\mathrm{L}x+1)=\mathrm{L}.x+\mathrm{L}a+\mathrm{L}a' \qquad =\mathrm{L}.x+r=\mathrm{L}.x+r$$

$$\mathrm{L}(x+2)=\mathrm{L}(x+1)+r+\mathrm{L}.a'+\mathrm{L}.a'' \qquad =\mathrm{L}(x+1)+$$

$$r'=\mathrm{L}(x+1)+r+r\,{}^{(1)}{}'$$

$$\mathrm{L}(x+3)=\mathrm{L}(x+2)+r'+r\,{}^{(1)}{}'+\mathrm{L}.a''+\mathrm{L}.a'''=\mathrm{L}(x+2)+$$

$$r''=\mathrm{L}(x+2)+r'+r\,{}^{(2)}{}'$$

$$\mathrm{L}(x+4)=\mathrm{L}(x+3)+r''+r\,{}^{(2)}{}'+\mathrm{L}.a''+2\mathrm{L}.a'''=\mathrm{L}(x+3)+$$

$$r'''=\mathrm{L}(x+3)+r''+r\,{}^{(3)}{}''$$

$$\mathrm{L}(x+5)=\mathrm{L}(x+4)+r'''+r\,{}^{(3)}{}''+\mathrm{L}.a''+3\mathrm{L}.a'''=\mathrm{L}(x+4)+$$

$$r''''=\mathrm{L}(x+4)+r'''+r\,{}^{(4)}{}'''$$

&c. &c.

Where it is plain that each logarithm is found by four additions r', r'', r''', &c. being got by two each.
$\quad{}^{(1)}\quad{}^{(2)}\quad{}^{(3)}$

Prop. V.

11. *To construct a Table of Logarithms by means of interpolation from the converging expressions* $\mathrm{L}.s$, $\mathrm{L}.s'$, $\mathrm{L}.s''$, &c.

If we consider the formation of equations (d), we easily perceive that the terms of $\mathrm{L}(x+n)$ observe the same law,

with respect to those of $L(x+n-1)$, which we observed in equations (c); we have therefore by a similar elimination,

$$\begin{aligned}
L.x &= L(x-2)+L.s\\
L(x+1) &= L(x-1)+L.s+L.s'\\
L(x+2) &= L.x+L.s+2L.s'+L.s''\\
L(x+3) &= L(x+1)+L.s+3L.s'+3L.s''+L.s'''\\
&\cdots\cdots\cdots\cdots\cdots\cdots\\
L(x+n) &= L(x+n-2)+L.s+nL.s'+\tfrac{n(n-1)}{1.2}L.s''+
\end{aligned}$$

which in order that the logarithms may be got from one another by addition, must be transformed as in the last proposition by the assumption of ρ, ρ', ρ'', &c.; $\rho_{(1)}$, $\rho'_{(1)}$, $\rho''_{(1)}$, &c.; $\rho_{(2)}$, $\rho'_{(2)}$, $\rho''_{(2)}$, &c.: thus if the case only requires us to use $L.s$, $L.s'$, and $L.s''$, make

$$\begin{aligned}
L.s+L.s' &= \rho\\
L.s+2L.s'+L.s'' &= \rho+L.s'+L.s'' = \rho'\\
L.s+3L.s'+3L.s'' &= \rho'+L.s'+2L.s'' = \rho''\\
&\text{\&c.} \qquad\qquad \text{\&c.} \qquad\qquad \text{\&c.}
\end{aligned}$$

by substituting which our equations become

$$\begin{aligned}
L.x &= L(x-2)+L.s\\
L(x+1) &= L(x-1)+L.s+L.s' = L(x-1)+\rho\\
L(x+2) &= L.x+\rho+L.s'+L.s'' = L.x+\rho'\\
L(x+3) &= L(x+1)+\rho'+L.s'+2L.s'' = L(x+1)+\rho''\\
&\text{\&c.} \qquad\qquad \text{\&c.} \qquad\qquad \text{\&c.}
\end{aligned}$$

If now we put successively 0, 1, 2 for n in the value of $L.s''\ldots\ldots^{n}$, given in Art. 7, we find

$$L.s = L\left(\frac{x}{x-2}\right);\ L.s' = L\left(\frac{(x+1)(x-2)}{x(x-1)}\right);\ L.s'' = L\left\{\frac{(x+2)(x-1)^2}{(x+1)^2(x-2)}\right\};\ \text{or}$$

$$L.s = 2M\left\{\frac{1}{x-1}+\frac{1}{3}\left(\frac{1}{x-1}\right)^3+\frac{1}{5}\left(\frac{1}{x-1}\right)^5+\text{\&c.}\right\}$$

$$\mathrm{L}.s' = -2\mathrm{M}\left\{\frac{1}{x^2-x-1}+\frac{1}{3}\left(\frac{1}{x^2-x-1}\right)^3+\frac{1}{5}\left(\frac{1}{x^2-x-1}\right)^5+\&c.\right.$$

$$\left.\mathrm{L}.s'' = \ 2\mathrm{M}\left\{\frac{2}{x^2-3x}+\frac{1}{3}\left(\frac{2}{x^2-3x}\right)^3+\frac{1}{5}\left(\frac{2}{x^2-3x}\right)^5+\&c.\right\}\right.$$

(BORDA's Series.)

PROP. VI.

12. *To expand* $\mathrm{L}.\alpha^{n\ldots n}$ *and* $\mathrm{L}.s^{n\ldots n}$ *into series of monomials of the form* $\frac{\mathrm{A}}{x^r}$.

$$\mathrm{L}(x+n) = \mathrm{L}.x+\mathrm{M}\left\{\frac{n}{x}-\frac{n^2}{2x^2}+\frac{n^3}{3x^3}-\ldots\ldots\ldots\pm\frac{n^r}{rx^r}\mp\right\}$$

$$-\frac{n+1}{1}\mathrm{L}(x+n-1) = -\frac{n+1}{1}\mathrm{L}.x+\mathrm{M}\left\{-\frac{n+1}{1}\cdot\frac{n-1}{x}+\frac{n+1}{1}\cdot\frac{(n-1)^2}{2x^2}\right.$$

$$\left.-\ldots+\frac{n+1}{1}\cdot\frac{(n-1)^r}{rx^r}\pm\right\}$$

$$\frac{(n+1)n}{1.2}\mathrm{L}(x+n-2) = \frac{(n+1)n}{1.2}\mathrm{L}.x+\mathrm{M}\left\{\frac{(n+1)n}{1.2}\cdot\frac{n-2}{x}-\frac{(n+1)n}{1.2}\cdot\frac{(n-2)^2}{2x^2}\right.$$

$$\left.+\ldots\pm\frac{(n+1)n}{1.2}\cdot\frac{(n-2)^r}{rx^r}\mp\right\}$$

&c. &c.

These added together will give $\mathrm{L}.\alpha^{n\ldots n}$. It is easy to see that $\mathrm{L}.x$ will disappear, because its coefficient $=(1-1)^{n+1}$; we have then, putting Σ to represent the sum of the terms formed by the different values of r,

$$\mathrm{L}.\alpha^{n\ldots n} = \mp\mathrm{M}.\Sigma\left\{\frac{n^r-\frac{n+1}{1}(n-1)^r+\frac{(n+1)n}{1.2}(n-2)^r-\frac{(n+1)n(n-1)}{1.2.3}(n-3)^r+}{rx^r}\right\}$$

where for r we are to take every whole number from one upwards; thus

$$\mathrm{L}.\alpha = \ \mathrm{M}\left\{\frac{1}{x}+\frac{1}{2x^2}+\frac{1}{3x^3}+\ldots\ldots\ldots\ldots+\frac{1}{rx^r}+\right\}$$

$$\left.\mathrm{L}.\alpha' = -\mathrm{M}\left\{\frac{1}{x^2}+\frac{1}{2x^4}+\frac{1}{3x^6}+\&c.\right.\right\}$$

&c. &c.

Hence $\mathrm{L}.\alpha'$ equals double the sum of the second, fourth, &c.

terms of L. a, with the sign changed. L. $s^{\prime\prime\ldots n}$ is expanded, by means of the logarithmic series, in a similar manner.

Prop. VII.

13. *To calculate a Table of Logarithmic sines, or cosines.*

It is quite evident, that, if, in the fractional products, in Propositions II, and III, instead of x, $x-1$, $x-2$, &c. we had used successively cos. x, cos. $(x-u)$, cos. $(x-2u)$, &c. or sin. x, sin. $(x-u$ sin. $(x-2u)$, &c., the reasonings made use of would have been equally applicable; and that the whole methods given in Proposition III, IV, V, including the general expressions for L. $a^{\prime\prime\ldots n}$, L$s^{\prime\prime\ldots n}$, (but not the expansions of the said expressions) will hold good here, after we have made the above mentioned substitutions. Thus if it is L. cos. x which we are calculating, we shall have

$$\text{L}.a=\text{L}\left(\frac{\cos.x}{\cos.(x-u)}\right);\ \text{L}.a'=\text{L}.\left(\frac{\cos.(x-u)\cos.(x+u)}{\cos.^2 x}\right);\ \text{L}.a''=\text{L}\left(\frac{\cos.(x+2u)\cos.^2 x}{\cos.(x-u)\cos.^2(x+u)}\right)$$

These are the logarithms of numbers converging continually towards unity, and must be found by the form for L. $\left(\frac{a}{b}\right)$.

XVI. *Two general propositions in the method of differences.* By Thomas Knight, *Esq.* *Communicated by* Taylor Combe, *Esq. Sec. R. S.*

Read February 27, 1817.

1. THOUGH so many ingenious writers have demonstrated, and, in various respects, extended the celebrated formulas of LA GRANGE, for $\Delta^n \varphi(x)$, $\Sigma^n \varphi(x)$ no one appears to have entertained the idea, that these, and the more general cases, in which the quantities under the functional sign have their differences variable, might be included in one simple form.

Mr. PRONY* is, I believe, the only mathematician who has given a form of any regularity to $\Delta^n \varphi(x)$, when the difference of x is variable; but he does not seem to have been aware of the capability of the method he was employing; and instead of embracing, as he might have done, all cases in one simple expression, he has proposed a formula which has neither any particular elegance in itself, nor any apparent relation to that which, in the simpler case, had been given by LA GRANGE.

I suppose the truth of the differential equations

$$\Delta^n \varphi = \varphi_n - \frac{n}{1}\varphi_{n-1} + \frac{n(n-1)}{1.2}\varphi_{n-2} - \frac{n(n-1)(n-2)}{1.2.3}\varphi_{n-3} + , \quad (1)$$

$$\Sigma^n \varphi = \varphi_{-n} + \frac{n}{1}\varphi_{-(n+1)} + \frac{n(n+1)}{1.2}\varphi_{-(n+2)} + \frac{n(n+1)(n+2)}{1.2.3}\varphi_{-(n+3)} + , \ldots (2):$$

where φ is any variable function whatever.

* LACROIX "Calc. des Diff." p. 25.

PROP. I.

2. *To find the* n^{th} *difference of a function of any number of variable quantities,* $\Delta^n \varphi(x, y, z, \&c.)$, *when the differences of* x, y, z, &c. *are any how variable.*

We will begin with a function of two variables;

Let $x_1 - x = u_1$, $x_2 - x = u_2$, $x_3 - x = u_3$, ... $x_n - x = u_n$;
$y_1 - y = w_1$, $y_2 - y = w_2$, $y_3 - y = w_3$, ... $y_n - y = w_n$;
} these values are simultaneous.

Let $\left(\frac{d^n \varphi(x,y)}{dx^n}\right) + \left(\frac{d^n \varphi(x,y)}{dx^{n-1} dy}\right) + \left(\frac{d^n \varphi(x,y)}{dx^{n-2} dy^2}\right) + \ldots\ldots + \left(\frac{d^n \varphi(x,y)}{dy^n}\right)$ be represented by $\Sigma\left(\frac{d^n \varphi(x,y)}{dx^{n-m} dy^m}\right)$; the sign Σ expressing *here* the sum of all the different values that will arise to the function within the brackets, by giving successively to m the values $0, 1, 2, 3, \ldots\ldots n$

Lastly, let the symbol $\boxtimes$ represent what may be called elective multiplication; thus $\Sigma\left(\frac{d^n \varphi(x,y)}{dx^{n-m} dy^m}\right) \boxtimes (u + w)^n$ will denote that each value of $\left(\frac{d^n \varphi(x,y)}{dx^{n-m} dy^m}\right)$ is to be multiplied by the corresponding term of the expanded binomial $(u+w)^n$; viz. $\left(\frac{d^n \varphi(x,y)}{dx^n}\right)$ by u^n, $\left(\frac{d^n \varphi(x,y)}{dx^{n-1} dy}\right)$ by $nu^{n-1}w$, $\left(\frac{d^n \varphi(x,y)}{dx^{n-2} dy^2}\right)$ by $\frac{n(n-1)}{1.2} u^{n-2} w^2$, and so on Then

$$\varphi(x+u_1, y+w_1) = \varphi(x,y) + \Sigma\left(\frac{d\varphi(x,y)}{dx^{1-m} dy^m}\right) \boxtimes (u_1 + w_1) + \frac{1}{2} \cdot \Sigma\left(\frac{d^2 \varphi(x,y)}{dx^{2-m} dy^m}\right) \boxtimes (u_1 + w_1)^2 + \frac{1}{2.3} \cdot \Sigma \cdot \left(\frac{d^3 \varphi(x,y)}{dx^{3-m} dy^m}\right) \boxtimes (u_1 + w_1)^3 +$$

$$\phi(x+u_2, y+w_2)=\phi(x,y)+\Sigma\left(\frac{d\phi(x,y)}{dx^{1-m}dy^m}\right)\boxtimes(u_2+w_2)+\frac{1}{2}\cdot$$
$$\Sigma\left(\frac{d^2\phi(x,y)}{dx^{2-m}dy^m}\right)\boxtimes(u_2+w_2)^2+\frac{1}{2.3}\cdot\Sigma\left(\frac{d^3\phi(x,y)}{dx^{3-m}dy^m}\right)\boxtimes(u_2+w_2)^3+$$
$$\phi(x+u_3, y+w_3)=\phi(x,y)+\Sigma\left(\frac{d\phi(x,y)}{dx^{1-m}dy^m}\right)\boxtimes(u_3+w_3)+\frac{1}{2}\cdot$$
$$\Sigma\left(\frac{d^2\phi(x,y)}{dx^{2-m}dy^m}\right)\boxtimes(u_3+w_3)^2+\frac{1}{2.3}\cdot\Sigma\left(\frac{d^3\phi(x,y)}{dx^{3-m}dy^m}\right)\boxtimes(u_3+w_3)^3+$$
&c. &c.

These values substituted in the equation $\Delta^n\phi=\phi_n-\frac{n}{1}\cdot\phi_{n-1}$
$+\frac{n(n-1)}{1.2}\cdot\phi_{n-2}-\frac{n(n-1)(n-1)}{1.2.3}\cdot\phi_{n-3}+$ give

$$\Delta^n\phi(x,y)=\phi(x,y)\left\{1-\frac{n}{1}+\frac{n(n-1)}{1.2}-\frac{n(n-1)(n-2)}{1.2.3}+\right\}$$
$$+\Sigma\left(\frac{d\phi(x,y)}{dx^{1-m}dy^m}\right)\boxtimes\left\{(u_n+w_n)-\frac{n}{1}(u_{n-1}+w_{n-1})\right.$$
$$\left.+\frac{n(n-1)}{1.2}(u_{n-2}+w_{n-2})-\frac{n(n-1)(n-2)}{1.2.3}(u_{n-3}+w_{n-3})+\right\}$$
$$+\frac{1}{2}\Sigma\left(\frac{d^2\phi(x,y)}{dx^{2-m}dy^m}\right)\boxtimes\left\{(u_n+w_n)^2-\frac{n}{1}(u_{n-1}+w_{n-1})^2\right.$$
$$\left.+\frac{n(n-1)}{1.2}(u_{n-2}+w_{n-2})^2-\frac{n(n-1)(n-2)}{1.2.3}(u_{n-3}+w_{n-3})^2+\right\}$$
$$+\cdot\frac{1}{2.3}\Sigma\left(\frac{d^3\phi(x,y)}{dx^{3-m}dy^m}\right)\boxtimes\left\{(u_n+w_n)^3-\frac{n}{1}(u_{n-1}+w_{n-1})^3\right.$$
$$\left.+\frac{n(n-1)}{1.2}(u_{n-2}+w_{n-2})^3-\frac{n(n-1)(n-2)}{1.2.3}(u_{n-3}+w_{n-3})^3+\right\}$$
&c. &c.

3. Now, by the general formula for the n^{th} difference of a variable quantity, (1),

$\Delta^n(x+y)^n=(x+y+u_n+w_n)^n-\frac{n}{1}(x+y+u_{n-1}+w_{n-1})^n$
$+\frac{n(n-1)}{1.2}(x+y+u_{n-2}+w_{n-2})^n-$ whence, making x and y vanish,

$\Delta^n(o+o')^n = (u_n + w_n)^n - \frac{n}{1}(u_{n-1} + w_{n-1})^n + \frac{n(n-1)}{1.2}(u_{n-2} + w_{n-2})^n -$;

the successive values of o being $u_1, u_2, u_3, \ldots\ldots u_n$

those of o' being $w_1, w_2, w_3, \ldots\ldots w_n$

observing therefore that $1 - \frac{n}{1} + \frac{n(n-1)}{1.2} - \frac{n(n-1)(n-2)}{1.2.3} +$

$= \Delta^n(o+o')^n$, and putting $\Sigma\left(\frac{d^{o+m}\varphi(x,y)}{dx^o dy^m}\right)$ for $\phi(x,y)$ our equation takes this form

$$\Delta^n\phi(x,y) = \Sigma\left(\frac{d^o\varphi(x,y)}{dx^{o-m}dy^m}\right)\boxtimes\Delta^n(o+o')^o + \Sigma\left(\frac{d^1\varphi(x,y)}{dx^{1-m}dy^m}\right)\boxtimes\Delta^n(o+o')^1 +$$

$$\frac{1}{2}\Sigma\left(\frac{d^2\varphi(x,y)}{dx^{2-m}dy^m}\right)\boxtimes\Delta^n(o+o')^2 + \frac{1}{2.3}\cdot\Sigma\left(\frac{d^3\varphi(x,y)}{dx^{3-m}dy^m}\right)\boxtimes\Delta^n(o+o')^3 + .$$

What has been done with respect to a function of two variables, the analyst will immediately see how to extend to a function containing any number; we may therefore without entering into any farther particulars, give the following

General Rule.

Let the successive values

of o be $u_1, u_2, u_3, \ldots\ldots u_n$

of o' be $w_1, w_2, w_3, \ldots\ldots w_n$

of o'' be $v_1, v_2, v_3, \ldots\ldots v_n$*

. . . &c. . . . . ,*then will*

$$\Delta^n\phi(x,y,z,\&c.) = \Delta^n\, e^{o+o'+o''+\&c.} \qquad (8)$$

provided that, after the expansion, we multiply every where a term of the form $A \times u^a_{n-r} \times w^b_{n-r} \times v^c_{n-r} \times$ &c. *by*

* Supposing $u_n - u = v_n$, &c. &c.

$$\left(\frac{d^{a+b+c+\&c.}\varphi(x,y,z,\&c.)}{dx^a.dy^b.dz^c.\&c.}\right).$$

4. The expression of LA GRANGE is a particular case of eq. (3), to perceive which we must observe that

$$\Delta^n e^{x+y+z+\&c.} = e^{x+y+z+\&c.} + u_n + w_n + v_n + \&c.$$

$$-\frac{n}{1}.e^{x+y+z+\&c.}+u_{n-1}+w_{n-1}+v_{n-1}+\&c.+$$

$$\frac{n(n-1)}{1.2}e^{x+y+z+\&c.}+u_{n-2}+w_{n-2}+v_{n-2}+\&c.+\&c.\text{ whence}$$

$$\Delta^n e^{o+o'+o''+\&c.} = e^{u_n+w_n+v_n+\&c.} - \frac{n}{1}e^{u_{n-1}+w_{n-1}+v_{n-1}}+\&c.$$

$$+\frac{n(n-1)}{1.2}.e^{u_{n-2}+w_{n-2}+v_{n-2}+\&c.}-\&c.;$$

which, if x, y, z, &c. have constant differences, or if u^n, u^{n-1}, u_{n-2}, &c. w_n, w_{n-1}, w_{n-2}, &c. v_n, v_{n-1}, v_{n-1} &c., &c. are nu, $(n-1)u$, $(n-2)u$, &c., nw, $(n-1)w$, $(n-2)w$, &c., nv, $(n-1)v$, $(n-2)v$, &c. &c. becomes

$$\left\{e^{u+w+v+\&c.}-1\right\}^n.$$

The equation (3) may be presented under another form; for if we compare the values of

$$\Delta^n e^{x+y+z+\&c.} \text{ and } \Delta^n e^{o+o'+o''+\&c.}, \text{ we see that}$$

$$\Delta^n e^{o+o'+o''+\&c.} = \frac{\Delta^n e^{x+y+z+\&c.}}{e^{x+y+z+\&c.}}, \text{ consequently}$$

$$\Delta^n \varphi(x,y,z,\&c.) = \frac{\Delta^n e^{x+y+z+\&c.}}{e^{x+y+z+\&c.}} \qquad (4)$$

where we must observe, with respect to the differential coefficients, the same rule as was given with eq. (3).

Prop. II.

5. *To find* $\Sigma^n \varphi(x, y, z, \&c.)$ *supposing that the differences of* x, y, z, &c. *are any how variable.*

We shall here make use of the same notation as we did in Prop. I, only let the preceding values

of x be $x-u_{-1}$, $x-u_{-2}$, $x-u_{-3}$, &c.
of y be $y-w_{-1}$, $y-w_{-2}$, $y-w_{-3}$, &c.
of z be $z-v_{-1}$, $z-v_{-2}$, $z-v_{-3}$, &c.
&c. &c.

It will be sufficient also to consider the case of two variable quantities, as was done in Prop. I.

First, we have, in general,

$$\varphi(x-u_{-r}, y-w_{-r}) = \varphi(x, y) - \Sigma\left(\frac{d\varphi(x,y)}{dx^{1-m}dy^m}\right) \boxtimes (u_{-r}+w_{-r}) +$$

$$+ \frac{1}{2} \cdot \Sigma\left(\frac{d^2\varphi(x,y)}{dx^{2-m}dy^m}\right) \boxtimes (u_{-r}+w_{-r})^2 - \frac{1}{2.3} \cdot \Sigma\left(\frac{d^3\varphi(x,y)}{dx^{3-m}dy^m}\right)$$

$\boxtimes (u_{-r}+w_{-r})^3 +$ which expression being combined with (s), putting for the sake of symmetry $\varphi(x, y) = \Sigma\left(\frac{d^0\varphi(x,y)}{dx^{0-m}dy^m}\right)$, gives

$$\Sigma^n \varphi(x, y) = \Sigma\left(\frac{d^0\varphi(x,y)}{dx^{0-m}dy^m}\right) \boxtimes \left\{(u_{-n}+w_{-n})^0 + \frac{n}{1} \cdot (u_{-(n+1)}\right.$$

$$\left. + w_{-(n+1)})^0 + \frac{n(n+1)}{1.2}(u_{-(n+2)}+w_{-(n+2)})^0 + \&c.\right\}$$

$$- \Sigma\left(\frac{d^1\varphi(x,y)}{dx^{1-m}dy^m}\right) \boxtimes \left\{(u_{-n}+w_{-n})^1 + \frac{n}{1} \cdot (u_{-(n+1)}\right.$$

$$\left. + w_{-(n+1)})^1 + \frac{n(n+1)}{1.2}(u_{-(n+2)}+w_{-(n+2)})^1 + \&c.\right\}$$

$$+\tfrac{1}{2}\cdot\Sigma\left(\frac{d^2\varphi(x,y)}{dx^{2-m}dy^m}\right)\boxtimes\left\{(u_{-n}+w_{-n})^s+\tfrac{n}{1}\cdot(u_{-(n+1)}+w_{-(n+1)})^s\right.$$
$$\left.+\tfrac{n(n+1)}{1.2}(u_{-(n+2)}+w_{-(n+2)})^s+\text{&c.}\right\}$$
$$+\text{&c.}$$

But by equat. (2) $\Sigma^n(x+y)^n=(x+y-u_{-n}-w_{-n})^n+\frac{n}{1}\cdot$
$(x+y-u_{-(n+1)}-w_{-(n+1)})^n+\frac{n(n+1)}{1.2}(x+y-u_{-(n+2)}-w_{-(n+2)})^n+$
whence

$$\Sigma^n(o+o')^n=\pm\left\{(u_{-n}+w_{-n})^n+\tfrac{n}{1}(u_{-(n+1)}+w_{-(n+1)})^n+\right.$$
$$\left.\tfrac{n(n+1)}{1.2}\cdot(u_{-(n+2)}+w_{-(n+2)})^n+\right\}$$

The upper or lower sign having place accordingly as m is even or odd. So that our equation may be expressed thus,

$$\Sigma^n\varphi(x,y)=\Sigma\left(\frac{d^0\varphi(x,y)}{dx^{0-m}dy^m}\right)\boxtimes\Sigma^n(o+o')^0+\Sigma\left(\frac{d^1\varphi(x,y)}{dx^{1-m}dy^m}\right)\boxtimes\Sigma^n(o+o')^1+$$
$$\tfrac{1}{2}\cdot\Sigma\left(\frac{d^2\varphi(x,y)}{dx^{2-m}dy^m}\right)\boxtimes\Sigma^n(o+o')^2+\tfrac{1}{2.3}\cdot\Sigma\left(\frac{d^3\varphi(x,y)}{dx^{3-m}dy^m}\right)\boxtimes\Sigma^n(o+o')^3+$$

and the analyst will, without any trouble, see the truth of the following rule for a function of any number of variables.

Let the preceding values

of o be $-u_{-1}, -u_{-2}, -u_{-3}, \ldots\ldots -u_{-n}$, &c.
of o' be $-w_{-1}, -w_{-2}, -w_{-3}, \ldots\ldots -w_{-n}$, &c.
of o'' be $-v_{-1}, -v_{-2}, -v_{-3}, \ldots\ldots -v_{-n}$, &c.
&c. &c.

then will

$$\Sigma^n\varphi(x,y,z,\text{&c.})=\Sigma^n\,e^{o+o'+o''+\text{&c.}} \qquad (5)$$

provided that, after expansion, we multiply a term of the form

$$A\times u^a_{-(n+r)}\times w^b_{-(n+r)}\times v^c_{-(n+r)}\times\text{&c. by}$$
$$\left(\frac{d^{a+b+c+\text{&c.}}\varphi(x,y,z,\text{&c.})}{dx^a\times dy^b\times dz^c\times\text{&c.}}\right)$$

We have by form (2),

$$\Sigma^n e^{x+y+\&c.} = e^{x+y+z+\&c.} {}_{-u} {}_{-w} {}_{-n} {}_{-v} {}_{-n} {}^{-\&c.} +$$
$$\frac{n}{1} \cdot e^{x+y+z+\&c.} {}_{-u} {}_{-(n+1)} {}^{-w} {}_{-(n+1)} {}^{-v} {}_{-(n+1)} {}^{-\&c.} +$$
$$\frac{n(n+1)}{1.2} \cdot e^{x+y+z+\&c.} {}_{-u} {}_{-(n+2)} {}^{-w} {}_{-(n+2)} {}^{-v} {}_{-(n+2)} {}^{-\&c.} +$$
&c. . . . . so that

$$\Sigma^n e^{o+o'+o''+\&c.} = e^{-u} {}_{-n} {}^{-w} {}_{-n} {}^{-v} {}_{-n} {}^{-\&c.} +$$
$$\frac{n}{1} \cdot e^{-u} {}_{-(n+1)} {}^{-w} {}_{-(n+1)} {}^{-v} {}_{-(n+1)} {}^{-\&c.} + \quad (6)$$
$$\frac{n(n+1)}{1.2} \cdot e^{-u} {}_{-(n+2)} {}^{-w} {}_{-(n+2)} {}^{-v} {}_{-(n+2)} {}^{-\&c.} + \&c.$$

By comparing these expressions, it appears that

$$\Sigma^n e^{o+o'+o''+\&c.} = \frac{\Sigma^n e^{x+y+z+\&c.}}{e^{x+y+z+\&c.}} \text{ and consequently, that}$$

$$\Sigma^n \varphi(x, y, z, \&c.) = \frac{\Sigma^n \cdot e^{x+y+z+\&c.}}{e^{x+y+z+\&c.}}, \text{ provided, \&c.}$$

It is scarcely necessary for me to observe, that the second member of the equation marked (6) becomes $\left\{ e^{u+w+v+\&c.} {}_{-1} \right\}^{-n}$ in the case of constant differences of x, y, z, &c.; for u_{-n}, $u_{-(n+1)}$, &c. become in this case, nu, $(n+1)\,n$, &c., and the w, s and v, s undergo a similar change.

The results of the preceding propositions may be brought into a very small compass, viz.

Δ^{-n} *representing* Σ^n, *the* n$^{\text{th}}$ *difference or the* n$^{\text{th}}$ *integral of a function of any number of variable quantities, and varying in any possible manner, will be expressed by the equation* $\Delta^n \varphi(x, y, z, \&c.) = \frac{\Delta^n e^{x+y+z+\&c.}}{e^{x+y+z+\&c.}}$; *provided that after expansion, we multiply* &c. &c. &c.

6. SCHOLIUM.

We may find, in many cases, very elegant and regular expressions for $\Delta^n\phi(x)$, by supposing $\phi(x+u)$ to be expanded differently from the form given by TAYLOR's theorem: as, for instance,

If $\phi(x+u)=\psi(x)+X.\chi(u)+X'\chi'(u)+X''\chi''(u)+$, (7) *where* X, X′, X″, &c. *represent any functions whatever of* x, and ϕ, ψ, χ, χ' &c. *any functions of the quantities they stand before, then* Δx, or u, *being constant,*

$$\Delta^n\phi(x)=X.\Delta^n.\chi(o)+X'.\Delta^n.\chi'(o)+X''.\Delta^n.\chi''(o)+\&c.:\text{ for}$$

$$\phi(x+nu)=\psi(x)+X.\chi(nu)+X'.\chi'(nu)+$$

$$-\tfrac{n}{1}.\phi(x+\overline{n-1}.u)=-\tfrac{n}{1}.\psi(x)-\tfrac{n}{1}X.\chi(\overline{n-1}.u)-\tfrac{n}{1}.X'.\chi'(\overline{n-1}.u)-$$

$$+\tfrac{n(n-1)}{1.2}\phi(x+\overline{n-2}.u)=\tfrac{n(n-1)}{1.2}\psi(x)+\tfrac{n(n-1)}{1.2}.X.\chi(\overline{n-2}.u)+\tfrac{n(n-1)}{1.2}.X'.\chi'(\overline{n-2}).u)+$$

&c. &c.

and because $\Delta^n.\phi=\phi_n-\tfrac{n}{1}\phi_{n-1}+\tfrac{n(n-2)}{1.2}\phi_{n-2}-$, this being added give

$$\Delta^n.\phi(x)=X.\Delta^n.\chi(o)+X'.\Delta^n.\chi'(o)+X''.\Delta^n.\chi''(o)+ \quad (8)$$

If form (7) soon terminates, the expressions for the differences are very simple, as in

Ex. 1. Sin. $(x+u)=$ sin. x cos. $u+$ cos. x, sin. u, which being compared with (7) gives $X=$ sin. x, $X'=$ cos. x, X'', &c. $=o$; $\chi(o)=$ cos. $o, \chi'(o)=$ sin. $o, \chi''(o)$, &c. $=o$; whence

$$\Delta^n.\text{ sin. } x=\text{sin. } x.\ \Delta^n.\text{ cos. } o+\text{cos. } x.\ \Delta^n.\text{ sin. } o.$$

Ex. 2. Tang. $(x+u)=$tang. $x+$ sec.$^2x\{$tang. $u+$tang. $x.$

tang.^{2}u + tang.^{2}x. tang.^{3}u + $\}$, see DELAMBRE, Preface to BORDA, p. 48, whence, by our expression,

Δ^{n}. tang. x = sec.$^{2}x\{\Delta^{n}$tang. o + tang. x . Δ^{n}. tang.^{2}o + tang.^{2}x. Δ^{n}. tang.^{3}o + $\}$

Ex. 8. L. sin. $(x + u)$ = L. sin. x + L. cos. u + M $\{$ Cot. x. tang. u — $\frac{1}{2}$cot.^{2}x. tang.^{2}u + $\frac{1}{3}$cot.^{3}x tang.^{3}u + $\}$ DELAMBRE, Preface to BORDA, p. 45; comparing with (7) and (8) we find Δ^{n} . L . sin . x = Δ^{n} . L . cos . o + M $\{$ Cot.$x\Delta^{n}$.tang.o — $\frac{1}{2}$cot^{2}.$x\Delta^{n}$.tang.^{2}o + $\frac{1}{3}$cot.^{3}x.Δ^{n}.tang.^{3}o — $\}$

In like manner, because

L . cos . $(x+u)$ = L . cos. x + L . cos. u — M $\{$ tang. x. tang. u + $\frac{1}{2}$ tang.^{2}x. tang.^{2}u + $\frac{1}{3}$ tang.^{3} x. tang.^{3}u + $\}$

Δ^{n} . L . cos. x = Δ^{n} . L . cos. o — M $\{$ tang. $x\,\Delta^{n}$. tang. o + $\frac{1}{2}$ tang.^{2} $x\,\Delta^{n}$. tang.^{2}o + $\frac{1}{3}$ tang.$^{3}x\,\Delta^{n}$ tang.^{3}o + $\}$

Then, because L . tang. = L . sin. — L . cos., Δ^{n}. L . tang. = Δ^{n} . L . sin. — Δ^{n} . L . cos., therefore

Δ^{n}. L . tang. x = M $\{$ (Cot. x + tang. x) Δ^{n}. tang. o — $\frac{1}{2}$ (cot.^{2}x — tang.^{2}x) Δ^{n}. tang.^{2}o + $\frac{1}{3}$ (cot.^{3}x + tang.^{3}x) Δ^{n}. tang.^{3}o — $\}$

If these forms were to be used for interpolation, we should have to calculate, before the commencement of a Table, Δ L . cos. o, Δ^{2}. L . cos. o, &c.; Δ . tang. o, Δ^{2} . tang. o, &c.; Δ . tang.^{2}o, Δ^{2}. tang.^{2}o, &c., &c. These latter quantities are to be multiplied by M, and will then serve for calculating the whole Table.

If three differences are sufficient, we have, making u=1′,*

* It is the decimal division of the circle which is supposed here.

Δ . L . cos. o = —, $0'53579$, Δ^2 . L . cos. o = —, $0'107158$, Δ^3 . L . cos. o =, $0''3$;

Δ . tang. o =, $0'1570796339$, Δ^2 . tang. o =, $0''78$, Δ^3. tang. o =, $0''78$;

Δ . tang.^{2} o =, $0'246740$, Δ^2. tang.^{2} o =, $0'493480$, Δ^2. tang.^{2} o = o, $0''3$;

Δ . tang.^{3} o =, $0''39$, Δ^2 tang.^{3} o =, $0''''232$, Δ^3. tang.^{3} o =, $0''''232$.

Suppose, for a particular example, we want the first three differences of L. sin. 50°, we have

Δ L . sin. 50° = —, $0'53579 + M\{,0'1570796339 -, 0'128370 +, 0''13\}$

Δ^2 L. sin. 50° = —, $0'107158 + M\{,0''78 -, 0'246740 +, 0''77\}$

Δ^3. L . sin. 50° = $M\{, 0''78 +, 0''77\}$ or

Δ L . sin. 50° =, 0000682081030, Δ^2 . L . sin . 50° = —, 0000000214249,

Δ^3. L . sin. 50° =, 0000000000068.

XVII. *Note respecting the demonstration of the binomial theorem inserted in the last volume of the Philosophical Transactions.* By Thomas Knight, *Esq.* *Communicated by* Taylor Combe, *Esq. Sec. R. S.*

Read April 17, 1817.

In looking into Mr. Spence's ingenious "Essay on Logarithmic Transcendents," a work published in 1809, but which I have been so unfortunate as never to have seen till within the last fortnight, I was not a little surprised to find that a demonstration of the binomial theorem, similar to the one I had the honour to present to the Royal Society, had been already given by that writer. The same may be said of the first proposition of the preceding Paper on the construction of Logarithms.

Having made this acknowledgment, I shall perhaps be pardoned for observing, that Mr. Spence is not particularly happy in the manner of developing the kind of functions he treats of in his preface. I shall endeavour to give the solution of a class of equations of which he (Pref. p. vii.) has considered a particular case: with this we will begin.

It is proposed to develope the function which has this property, viz.

$$\phi(1+x)+\phi(1+y)+\phi(1+z)=\phi(1+x.1+y)+\phi(1+x.1+z)+\phi(1+y.1+z)-\phi(1+x.1+y.1+z).$$

Assume $\phi(1+x)=A+A'x+A''x^2+A'''x^3+\ldots\ldots A^{n}\ldots\ldots x^n+$, and after making the requisite substitutions in the given

equation, we see immediately that $A=o$, and that A' and A'' are arbitrary. Then, to find the law of the coefficients, in this and other similar cases, where there are any number of independent quantities, $x, x', x'', \ldots\ldots x^{n-1}$, transpose all the equation to one side, and find the coefficient of the first power of x^{n-1}, then the coefficient of the first power of x^{n-2} in the former coefficient, then again, in this last, the coefficient of the first power of x^{n-3}: and having arrived in this manner at the coefficient of x', it will have the form $a+bx+cx^2+\ldots\ldots+rx^n+$, and the equation $r=o$ will give the law sought for.

In the present case, putting $\rho=x+(1+x)z$, $\sigma=1+x+(1+x)z$, we find, by equalling to o the coefficient of the first power of y,

$$o=\left.\begin{matrix}A'+2A''\\ +A'\end{matrix}\right\}(x+z)+\left.\begin{matrix}3A'''\\ 2A''\end{matrix}\right\}(x^2+z^2)+\ldots\ldots+\left.\begin{matrix}(n+1)A^{n-1}\\ nA^{n}\end{matrix}\right\}(x^n+z^n)+$$
$$-A'\sigma-2A''\sigma\rho-3A'''\sigma\rho^2-\ldots\ldots-(n+1)(A^{n-1}\sigma\rho^n-$$

If in this we equal to o the coefficient of the first power of z, there arises

$$o=2A''-A'x-2A''(1+3x+2x^2)$$
$$-3A'''(2x+5x^2+3x^3)$$
$$-4A^{iv}(3x^2+7x^3+4x^4)$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$-nA^{n-1}(\overline{n-1}\,.\,x^{n-2}+\overline{2n-1}\,.\,x^{n-1}+nx^n)$$
$$-\ \&c.\ \ldots\ldots\ldots\ldots\ldots$$

whence we find for the general law of the coefficients $(n>2)$,

$$n(n-1)A^{n-1}+\overline{2n-3}\,.\,\overline{n-1}\,.\,A^{n-2}+(n-2)^2A^{n-3}=o\ldots\ldots(1)$$

From which let us suppose that we have calculated a few of the coefficients, and arrived at the result of Mr. SPENCE, viz.

$\varphi(1+x)=A'(x-\frac{x^2}{2.3}+\frac{3x^4}{2.3.4}-\ldots)+A''L^2(1+x)\ldots\ldots(2)$

nothing can be easier than to find the value of the remaining series; for it is quite obvious, from the equation expressing the property of the function, that $L(1+x)$ is a particular value of $\varphi(1+x)$. The same also appears from equation (1), in which, if we put $A''^{\ldots(n-2)}=\frac{1}{n-2}$, $A''^{\ldots(n-1)}=\frac{-1}{n-1}$, $A'^{\ldots n}=\frac{1}{n}$, the left hand member vanishes.

Make then, in equation (2), $A'=1$, $A''=\frac{-1}{2}$, and it becomes $L(1+x)=x-\frac{x^2}{2.3}+\frac{3x^4}{2.3.4}-$ &c. $-\frac{1}{2}L^2(1+x)$, whence $x-\frac{x^2}{2.3}+\frac{3x^4}{2.3.4}-$ &c. $=L(1+x)+\frac{1}{2}L^2(1+x)$, and finally

$\varphi(1+x)=A'\left\{L(1+x)+\frac{1}{2}L^2(1+x)\right\}+A''L^2(1+x)$

Let us now endeavour to develope the function next in order of the same class, viz. $\varphi(1+x)$, having the following property,

$\varphi(1+w)+\varphi(1+x)+\varphi(1+y)+\varphi(1+z)=\varphi(1+x.1+y)+\varphi(1+x.1+z)+\varphi(1+y.1+z)+\varphi(1+w.1+x)+\varphi(1+w.1+y)+\varphi(1+w.1+z)-\ldots\ldots(3)$

$\varphi(1+x.1+y.1+z)-\varphi(1+w.1+y.1+z)-\varphi(1+w.1+x.1+z)-\varphi(1+w.1+x.1+y)+\varphi(1+w.1+x.1+y.1+z)$.

Assume $\varphi(1+x)=A+A'x+A''x^2+\ldots\ldots+A''^{\ldots n}x^n+$ and make the requisite substitutions; we shall find $A=0$, A', A'' and A''' arbitrary. Then to have the law of the coefficients, find, according to the rule, the coefficient of x in the coefficient of z in the coefficient of y, and, comparing in this the coefficients of the powers of w, we find

$n(n-1)(n-2)A''^{\dots n}+3(n-1)(n-2)^2.A''^{\dots(n-1)}+(3.\overline{n-2}^2.n-3$
$+n-2)A''^{\dots(n-2)}+(n-2.n-3.n-4+n-3)A''^{\dots(n-3)}=0$

If we take successively 4, 5, 6, &c. for n we find

$$\varphi(1+x)=A'\left\{x-\tfrac{1}{24}x^4+\tfrac{9}{120}x^5-\right\}$$
$$+A''\left\{x^2-\tfrac{7}{12}x^4+\tfrac{11}{12}x^5-\right\}$$
$$+A'''\left\{x^3-\tfrac{3}{4}x^4+\tfrac{7}{4}x^5-\right\}$$

The series at bottom is $L^3(1+x)$; the series next above it is $L^2(1+x)+L^3(1+x)$. To find the upper series we have the same means as in the last Problem, $L(1+x)$ being a particular value of $\varphi(1+x)$; make then $A'=1$, $A''=-\frac{1}{2}$, $A'''=\frac{1}{3}$; our equation becomes $L(1+x)=x-\frac{1}{24}x^4+\frac{9}{120}x^5-$

$$-\tfrac{1}{2}\left\{L^2(1+x)+L^3(1+x)\right\}$$
$$+\tfrac{1}{3}L^3(1+x)$$

whence $x-\frac{1}{24}x^4+\frac{9}{120}x^5-$ &c. $=L(1+x)+\frac{1}{2}L^2(1+x)$
$+\frac{1}{6}L^3(1+x)$, and

$$\varphi(1+x)=A'\left\{L(1+x)+\tfrac{1}{2}L^2(1+x)+\tfrac{1}{6}L^3(1+x)\right\}$$
$$+A''\left\{L^2(1+x)+L^3(1+x)\right\}$$
$$+A'''\ L^3(1+x)$$

which is the complete solution of the proposed equation.

As, however, this result has been obtained by the inspection of only a few terms of two series, a doubt may be entertained with respect to its truth: make therefore $w=x=y=z$, in equation (3), it will become $4\varphi(1+x)=6\varphi(1+x)^2-4\varphi(1+x)^3+\varphi(1+x)^4$, which by the substitution of the

value found above for $\varphi(1+x)$ is found to be identical, and the truth of the solution is proved.

We may now attempt the solution of the general problem, viz. Let $x, x', x'', x''', \ldots x^{(n-1)}$ be independent quantities; it is required to find $\varphi(1+x)$ from the following equation,

$$\Sigma.\varphi(1+x^{(m)}) = \Sigma.\varphi(1+x^{(m)}.1+x^{(m)}) - \Sigma.\varphi(1+x^{(m)}.1+x^{(m)}.1+x^{(m)}) + \Sigma.\varphi(1+x^{(m)}.1+x^{(m)}.1+x^{(m)}.1+x^{(m)}) - \Sigma.\varphi(1+x^{(m)}.1+x^{(m)}.1+x^{(m)}.1+x^{(m)}.1+x^{(m)}) + \&c.$$

Assume as usual $\varphi(1+x) = A + A'x + A''x^2 + A'''x^3 + \ldots$; but instead of attempting to find the law of the coefficients, we may easily convince ourselves that $\varphi(1+x)$ will have the following form, viz.

$$\varphi(1+x) = A' \left\{ L(1+x) + \alpha'' L^2(1+x) + \alpha''' L^3(1+x) + \alpha'''' L^4(1+x) + \ldots\ldots + \alpha^{(n-1)} L^n(1+x) \right\}$$
$$+ A'' \left\{ L^2(1+x) + \beta''' L^3(1+x) + \beta'''' L^4(1+x) + \ldots\ldots + \ldots\ldots (4) \right.$$
$$\left. \beta^{(n-1)} L^n(1+x) \right\}$$
$$+ A''' \left\{ L^3(1+x) + \gamma'''' L^4(1+x) + \ldots\ldots + \gamma^{(n-1)} L^n(1+x) \right\}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ A^{(n-1)} L^n(1+x)$$

This form evidently includes, as particular solutions,

$$L(1+x), L^2(1+x), L^3(1+x), \ldots\ldots L^n(1+x):$$

and, by means of these particular solutions, we are enabled to find the coefficients α'', α''', α'''', &c. β'', β'''', &c., γ'''', &c. &c. For let

$$L(1+x) = x + b''x^2 + b'''x^3 + b''''x^4 + \ldots$$
$$L^2(1+x) = x^2 + c'''x^3 + c''''x^4 + \ldots$$
$$L^3(1+x) = x^3 + d''''x^4 + \ldots$$
$$\&c. \qquad \&c.$$

By changing A', A'', A''', &c. into the coefficients of each of these expressions, successively, we have p particular values of equation (4); viz.

$$L(1+x)=L(1+x)+\alpha''L'(1+x)+\alpha'''L^{3}(1+x)+\alpha^{iv}L^{4}(1+x)+ \ldots\ldots+\alpha^{r-1}L^{r}(1+x)$$

$$+b''\left\{L'(1+x)+\beta'''L^{3}(1+x)+\beta^{iv}L^{4}(1+x)+\ldots\ldots+\beta^{r-1}L^{r}(1+x)\right\}$$

$$+b'''\left\{L^{3}(1+x)+\gamma^{iv}L^{4}(1+x)+\ldots\ldots+\gamma^{r-1}L^{r}(1+x)\right\}$$

$$+b^{iv}\left\{L^{4}(1+x)+\ldots\ldots+\delta^{r-1}L^{r}(1+x)\right\}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+b^{r-1}\times L^{r}(1+x)$$

from which we derive the equations $b''+\alpha''=0$, $b'''+b''\beta'''+\alpha'''=0$, $b^{iv}+b'''\gamma'''+b''\beta^{iv}+\alpha^{iv}=0$, &c.; next we have

$$L'(1+x)=L'(1+x)+\beta'''L^{3}(1+x)+\beta^{iv}L^{4}(1+x)+\ldots\ldots+\beta^{r-1}L^{r}(1+x)$$

$$+c'''\left\{L^{3}(1+x)+\gamma^{iv}L^{4}(1+x)+\ldots+\gamma^{r-1}L^{r}(1+x)\right\}$$

$$+c^{iv}\left\{L^{4}(1+x)+\ldots\ldots+\delta^{r-1}L^{r}(1+x)\right\}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+c^{r-1}\times L^{r}(1+x)$$

whence we get the equations $c'''+\beta'''=0$, $c^{iv}+c'''\gamma^{iv}+\beta^{iv}=0$, &c.

The next particular solution is

$$L^{3}(1+x)=L^{3}(1+x)+\gamma^{iv}L^{4}(1+x)+\ldots\ldots+\gamma^{r-1}L^{r}(1+x)$$

$$+d^{iv}\left\{L^{4}(1+x)+\ldots\ldots+\delta^{r-1}L^{r}(1+x)\right\}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+d^{r-1}\times L^{r}(1+x)$$

whence $d^{iv}+\gamma^{iv}=0$, &c. and by proceeding in the same way we have as many equations as the coefficients which are to be determined.

So much for the expansion of these functions.

What Mr. SPENCE means by the note in page ix. of his Preface, where he speaks of the integral $\iint \frac{d^2x}{x} = \phi(x)$ an equation which is evidently impossible, I am unable to form the smallest conjecture.

Papcastle, March 3, 1817.

XVIII. *On the passage of the ovum from the ovarium to the uterus in women.* By Sir Everard Home, *Bart. V. P. R. S.*

Read May 1, 1817.

No subject connected with physiology has more employed the attention of the anatomist and philosopher than the first formation of the embryo in the class mammalia, and yet even at this day, when the same subject has been completely investigated in oviparous animals, and it is known that an ovum is formed in the ovarium of the quadruped, the circumstances respecting its impregnation have not been ascertained.

The great Harvey, although supplied by the munificence of his king with deer in all the different stages after being fit for the male, was unsuccessful. John Hunter, who prosecuted the same enquiry in the ewe, also failed. His brother, Dr. William Hunter, in his splendid work on the Gravid Uterus, has given the most correct representations of the human embryo from the end of the third week till the time of birth, but has not said any thing upon the subject of impregnation.

Haighton and Cruikshank, by experiments on rabbits, confirmed the opinion of De Graaf,* that an ovum is carried from the ovarium into the uterus, but by mistaking the corpus

* De Graaf's observations are mentioned in the 7th Volume of the Phil. Trans. p. 4052. In the same volume, p. 4018, Dr. Kerkringius's observations concerning eggs to be found in all sorts of females are noticed.

luteum for the effect of impregnation, instead of the substance in which the ovum is formed, which at that time was the generally received opinion, got entangled in theoretical opinions, which misled them in their farther enquiries.

In this state of our knowledge upon this most interesting subject, accident has done what no predetermined experiments had accomplished, it has enabled me to detect the ovum in the human uterus. It is so small, that had not the uterus been previously hardened in spirit as well as the ovum itself, it probably would have escaped observation, and, after it was found, it could not have been identified to be the ovum from which a child was to be produced, had it not been brought under the eye of Mr. BAUER, the only person, I may say, in this or any other country, who could so correctly apply to it the powers of the microscope, as to determine its form; could so separate its parts on the field of the microscope, as to display its organization; and so delineate what he saw, as to convey distinct notions that it was the first rudiments of a child.

I shall first give the history of the woman's case from the time of her impregnation, and then detail the appearances that were met with in the uterus and ovaria, after death.

A servant maid, 21 years of age, had been missing from her master's house, on the 7th of January 1817, for several hours in the forenoon; she came home in high spirits, said she had bought a pair of corsets and some other parts of dress.

In the evening, she got her fellow servant to assist her in putting on the corsets, but on lacing them she complained of being sick, and all over unwell; on taking some brandy she

recovered a little, and went to bed. Next day she was much the same; the period of menstruation had arrived, but it did not come on, and from this time there was a wildness in her manner, and she appeared distressed in her mind. On the 13th she had an epileptic fit attended with delirium, and on the 15th, about ten o'clock in the forenoon, died.

After death, the uterus showed signs of pregnancy, and from the statement that has been given, she appears to have been impregnated on the 7th of January, eight days before her death; for, although she was known to have a lover, there are circumstances to prove, that she could not have seen him after that time, nor for many days before.

The uterus having been hardened in spirit, with the assistance of Mr. CLIFT, I examined the parts. The right ovarium had a small torn orifice upon the most prominent part of its external surface, we slit it open in a longitudinal direction, in a line close to the edge of this orifice; the orifice was found to lead to a cavity filled up with coagulated blood, and surrounded by a yellowish organized structure. Upon opening into the cavity of the uterus, its inner surface was covered with an exudation of coagulable lymph, beautifully represented in the drawing; (Pl. VIII.), the ovum lay concealed among the long fibres of coagulable lymph near the cervix, and was brought to view by separating them with the point of a needle which I employed in making the search. As soon as it was disentangled, it rose up, moving along with the loose ends of the fibres into the spirit, by which the parts were covered. It had an oval appearance, one portion of it was quite white, the other semi-transparent; but soon after, being exposed to the spirit, the whole

became opaque. The os tincæ was entirely shut up with a strong solid jelly, the two orifices at the angles of the uterus, by which it communicates with the Fallopian tubes, were both pervious.

As the ovum was so extremely small as to admit of dispute, whether it was one or not, I carried it immediately to Kew to Mr. Bauer, who, after examining it, said that it looked like the egg of an insect. His drawings of the ovum and uterus, show to what an excellence microscopical observations can be carried, since in so small a particle of animal matter, he has pointed out the effects of impregnation even before any part of the vascular system had been formed, and where only the two projecting points within the ovum had been marked out as the future situations of the heart and brain. These two points are still to be distinguished in the ovum in a dried state, and that towards the broadest end is the largest.

Small as this ovum is, it bears a very fair proportion to that represented by Dr. Hunter at the end of three weeks; and, had this woman lived twenty-four hours longer, the ovum would probably have in that time been united to the fibrous structure surrounding it, and appeared secluded from the cavity of the uterus in the same degree as Dr. Hunter's is represented to be.

The corpus luteum has always been considered as the effect of impregnation, and a certain mark of conception having taking place, as I have already observed; but in this case there was not only the corpus luteum belonging to the present conception, but one still more distinct in the middle of the ovarium. This unexpected appearance of two corpora lutea, made me enquire farther into the subject, and led me to discover that the corpus luteum, in its origin, is a solid

compact glandular substance in which the ovum is formed, and, after the ovum is expelled, the blood which fills up the cavity is gradually absorbed, leaving a small cavity, which marks the place where the ovum had been.

Upon examining the ovaria of several women who had died virgins, and in whom the hymen was too perfect to admit of the possibility of impregnation, there were not only distinct corpora lutea, but also, as will be found in the present case, small cavities round the edge of the ovarium, evidently left by ova that had passed out at some former period, so that this happens during the state of virginity; and, as in Mr. CRUIKSHANK's experiments, the fimbriæ of the Fallopian tube of the rabbit in heat, were found embracing the ovarium, although she had not received the male, we cannot doubt, that every time a female quadruped is in heat, one or more ova pass from the ovarium to the uterus, whether she receives the male or not.

These facts explain the error which physiologists have gone into, of mistaking the corpus luteum, in which another ovum is forming, for that which belonged to the ovum of the present conception, and which at the time of delivery has disappeared.

Mr. BAUER's drawings not only show the changes which take place in the ovarium, for the purpose of forming the ova, but also the internal surface of the Fallopian tube at the time the ovum passes along it in its course to the uterus, which I believe has never before been represented.

The appearances are so clearly shown in the drawings, that it is not necessary to describe them: I shall therefore confine myself to an explanation of their probable uses.

The dilatation of this tube, at a small distance from the

fimbriæ, appears to be fitted for the reception of the ovum as well as of the semen, and the ovum is probably retained in this situation for several days, to prolong the opportunity of its being impregnated.

It has been disputed, whether the semen ever comes in contact with the ovum, or even arrives at the uterus; but as Mr. HUNTER has proved by experiment that it reaches the uterus,* and as there is no impediment to its passage from that organ to the ovarium, it must be admitted that the semen reaches the ovum before impregnation can take place.

The formation of ova in the ovaria, and their appearing in that organ in succession, joined to the circumstance of animals during the warm season being ready to receive the male once a month, leads to an opinion very contrary to that which is commonly received respecting menstruation. This discharge has been supposed a previous step, preparing the uterus for uterogestation; and if a woman has not been impregnated soon after menstruation, it is presumed that she may be more fortunate after her next period.

It is clear from the case which has been stated, that such periods are totally unconnected with the formation of the ovum, the process of its leaving the ovarium, or its impregnation; but, if impregnation does not take place, such a discharge may be necessary for the relief of parts to which there had been so great a derivation of blood, as the only means of restoring them to their natural state. The uterus in women and in the monkey has a more compact form than in other animals, which may explain the circumstance of menstruation being confined to them.

* J. HUNTERUS canis fœminæ inter coeundum occisæ, uterum aperuit; quo facto maris semen in ipsum uterum, per saltus intromissum, clare vidit.

In proof of menstruation not being necessary for impregnation, I shall mention the following case. A young woman was married before she was seventeen, and, although she had never menstruated, became pregnant; four months after her delivery she became pregnant a second time, and four months after the second delivery she was a third time pregnant, but miscarried; after this she menstruated for the first time, and continued to do so for several periods, and again became pregnant.

I have given Mr. BAUER's account of the ovum, and the drawings he has made of it, in his own words, than which none can be more clear or satisfactory.

"On closely examining the subject under the microscope, I found it consisted of membrane, which, considering the extreme minuteness of the subject, is of considerable thickness and consistence, very little transparent, quite smooth, and milk white, forming a kind of bag or pouch of an irregular oval shape, not quite $\frac{1}{100}$ parts of an inch in length, and in its middle about $\frac{1}{100}$ parts of an inch broad; on one side it has an elevated ridge or large fold along the whole length, and on the opposite side it is open nearly the whole length, but has no appearance of being torn, the edges of the membrane being smoothly rolled inwards, which gives it much the shape of a little shell of the genus Voluta.

"When laid on glass, the membrane admitted easily to be laid open on both sides, with the point of a fine camel hair pencil. When thus opened, I found it contained another smaller bag somewhat less than $\frac{1}{100}$ parts of an inch long, and not quite $\frac{1}{100}$ parts of an inch broad, ending at the upper extremity nearly in a point, but the under extremity was very obtuse or truncate, and in the middle it was slightly

contracted, which gave it the appearance of a young seed capsule of some plants that contain only two seed kernels.

"This inner bag consisted of a seemingly very thin, perfectly smooth, and glossy membrane, which seemed to have considerable strength, as it bore to be rubbed pretty strongly, not only with the camel hair pencil, but also with the point of the quill; it seemed to be filled with some thick slimy substance, as an impression made on it with the point of the quill remained for a considerable time visible; it contained two round corpuscules, apparently more opaque, and of a yellowish tint; they were not only visible through the transparent membrane, but they swelled the membrane over them, so that the light and shade made them to be distinctly seen; and by slightly pressing the bag with the quill between the two corpuscules, they could be separated to a greater distance from each other, but on putting more moisture upon the subject, they returned quickly to their former position. This little bag was along its whole length, with its back part strongly fixed to the outer membrane, at least I could not remove it with the camel hair pencil, and more force I was afraid to employ.

"I attempted to open the little bag, if possible, to extract the corpuscules, but on piercing, with the point of a very small needle, the upper extremity, a thick slimy matter, like honey, came out, and with the membrane adhered to the needle, so that I could no farther proceed; and fearful of spoiling the whole, I gave up the attempt, and left the subject on the glass to dry.; but I observed, as the spirit and moisture gradually evaporated, so the little bag flattened, and, as if melting, shrunk into the outer membrane, and almost disap-

peared, but in a strong light was still visible in the microscope.

"When quite dry, its colour changed to a light yellowish brown, and it lay quite loose on the glass, except at the upper extremity, where I attempted to open it; it was strongly glued to the glass, and it required several times to be moistened at that part with water, to remove it from the glass.

"I have now placed it between two pieces of talc in an ivory slider; and in a strong light the two corpuscules may still be seen through a common magnifying glass."

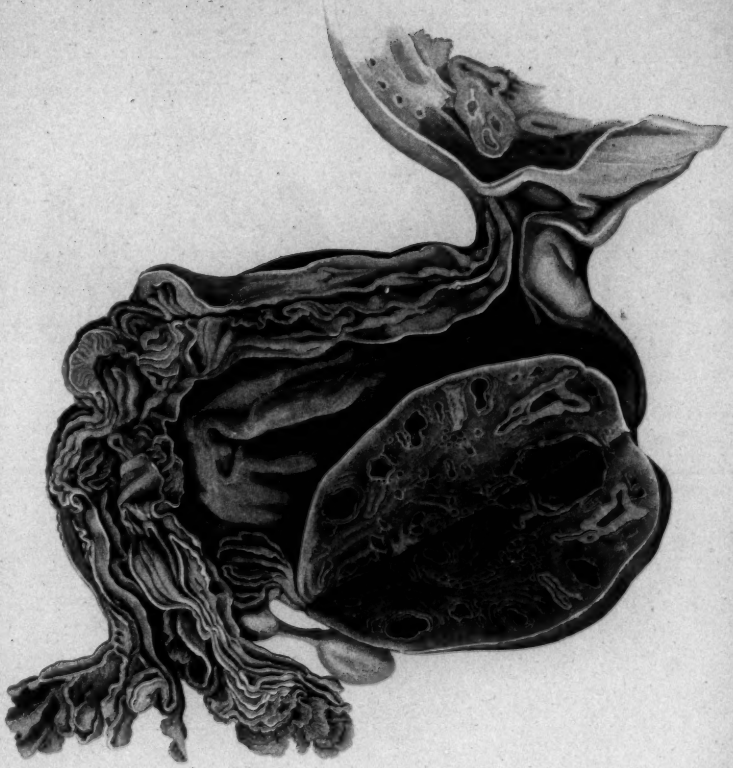
The drawing of the uterus (Pl. VIII.) is of the natural size: the parts are so distinct that no letters of reference appear to be necessary to point them out. The ovum is shown exactly in the spot in which it was discovered, with the appearance which it at that time put on.

The drawings of the ovaria and Fallopian tubes are magnified four times, to give a more exact notion, than could be otherwise done, of the canal through which the ovum passes, before it arrives at the cavity of the uterus. The appearance the corpora lutea put on, is the most exact representation from nature. In the right ovarium, cells remain where former ova had been formed, and one corpus luteum, which is cut through the middle, has made considerable advance in its formation, another appears to be in a much earlier stage, all the different orifices are the transverse and oblique sections of blood vessels.

In the left ovarium, the opening through which the ovum, the subject of the present Paper, passed out, is distinctly seen, and the cavity in which it was contained, is filled with coagulated blood in a laminated form; behind this, the glandular

Fran. Bauer del.

S. Basire sc.



Franz. Bauer del.

C. Basire sc.

Fig. 1.

Fig. 2.

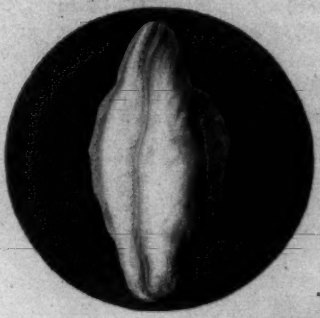
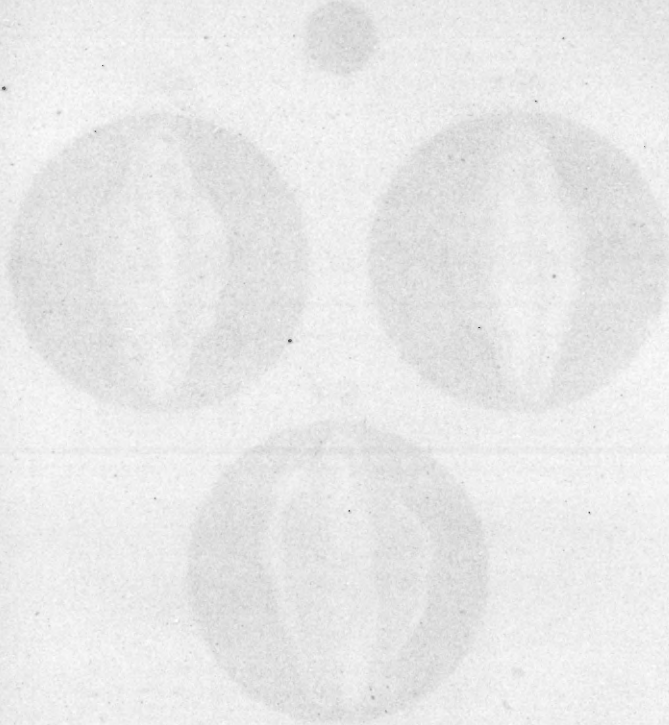


Fig. 3.

Fig. 4.

Franz Bauer del.

F. Basire sc.



structure of the corpus luteum is readily distinguished, of the form of an irregular oval. Towards the other end of this ovarium is the transverse section of a corpus luteum, not far enough advanced in its growth to have a cavity in which the future ovum is to be generated, the whole having one uniform glandular structure, the general form, like that of the other, being irregularly oval. The orange colour, peculiar to the glandular structure of the corpora lutea, which is very bright, and forms their distinguishing character, is not given, as the drawing is intended to be engraved.

Explanation of Plates VIII, IX, X, XI.

Plate VIII.

The uterus laid open from behind, showing the ovum in the situation in which it was discovered. The other parts require no description.

Plate IX.

The right ovarium laid open, showing the orifice at which the ovum escaped. The Fallopian tube laid open through its whole length. All these parts are minutely described in the Paper.

Plate X.

The left ovarium and Fallopian tube laid open.

Plate XI.

Different views of the ovum.

Fig. 1. The ovum represented of its natural size.

Fig. 2. The ovum magnified, exposing that side which is covered by one uniform membrane.

Fig. 3. The opposite side exposed, to show that the external membrane is disunited through its whole length, and unattached to the one under it.

Fig. 4. The two disunited edges of the outer membrane turned aside, exposing the inner membrane, through which are seen two very slight protuberances, the probable seat of the future heart and brain.

XIX. *Some farther observations on the use of the Colchicum Autumnale in Gout.* *By Sir* Everard Home, *Bart. V. P. R. S.*

Read May 8, 1817.

I LAID before the Society, some experiments and observations in favour of this medicine acting upon the gout through the medium of the circulation, and not by its effects directly upon the stomach and intestinal canal.

The object of the present Paper is to show that the infusion throws down a deposit, the separation of which does not appear to diminish the specific effects upon the gout, and renders those upon the stomach and intestines milder than when the deposit is taken along with the infusion.

The bulb of the Colchicum Autumnale contains a certain quantity of extractive matter, and a large portion of mucilage, both of which are taken up by the wine, in the first instance; when the strained liquor is allowed to stand, a considerable deposit almost immediately takes place.

In the first trials that were made with this medicine in St. George's Hospital, it was natural to enquire whether this deposit contained any medical virtues, and upon trials frequently repeated, it was found to have none.

This led to the opinion that the extractive matter suspended in the wine, was alone the active part of the medicine, and not only the first deposit was inert, but also that which from time to t me was afterwards found to take place.

Of this opinion I was led to entertain considerable doubts, in consequence of having found upon one occasion, in which I took half a bottle of the Eau Medicinale which had been poured off without shaking the bottle, that the sensible effects were very mild; those produced by the other half, in which the deposit was mixed, were unusually severe, the nausea being greater, and a greater number of stools being produced.

These doubts were much strengthened, when I found that the effects of the Eau Medicinale are more violent upon many stomachs than those of the vinous infusion of the Colchicum, which probably arises from the Eau Medicinale being kept in small bottles, in consequence of which all the deposit that takes place is given along with the infusion, while the vinous infusion of Colchicum being kept in large bottles, the deposit falls to the bottom. If such deposit increased the powers of the medicine in counteracting the symptoms of gout, it would be unnecessary to prosecute this investigation farther, since it would be absurd to diminish the violence of a medicine, if, by so doing, its efficacy is to be diminished in an equal degree.

To ascertain this point, I gave 60 drops of the vinous infusion of Colchicum, in which there was no deposit whatever, to a man labouring under a severe paroxysm of gout, to which he was a great martyr, and whose paroxysms were usually of several weeks continuance; he was 60 years of age.

The medicine was exhibited on the 17th of January 1817, his pulse being 115. In half an hour, he had slight nausea, which soon went off. In 5 hours, a profuse perspiration came on, and the pain of the gout entirely subsided, leaving a soreness in the parts that had been affected. In 12 hours, the bowels were gently moved, his pulse 105 and irregular;

in 14 hours, his bowels were acted on a second time; in 19 hours, his pulse was 92, and natural; in 48 hours, he was quite well, and has continued so, a period of more than three months.

The result of this case satisfied me, that the infusion contained the specific remedy for the gout, and that the deposit is not necessary for its removal.

This rendered it probable that, where the deposit is taken along with the infusion, its solid form prevents it from being carried into the circulation of the blood, and it remains in the stomach, producing more or less mischief in that viscus, without being any way concerned in driving away the disease for which the medicine was exhibited; in this respect resembling many of the salts of mercury, which irritate the bowels, without relieving the symptoms of the venereal disease.

I explained these opinions to Mr. GATCOMBE, who gives me his assistance in my professional pursuits, and requested him to investigate this subject.

To do this more completely, he began by repeating the three experiments detailed in my former Paper, substituting the Eau Medicinale for the vinous infusion of Colchicum, so as to determine with more precision, whether they are or are not the same medicine.

Exp. 1. Thirty drops of the Eau Medicinale with the deposit, were injected into the jugular vein of a dog: the effects were the same, as in my experiment with the same quantity of the vinous infusion of Colchicum, only the animal was 2 hours longer in recovering from them, and was purged for 9 hours afterwards.

Exp. 2. Sixty drops of the Eau Medicinale, were given by

the mouth to the same dog: the effect was less, than in my experiment with the vinous infusion of Colchicum exhibited in the same quantity: this arose from a very copious evacuation of urine having been produced.

Exp. 3. One hundred and sixty drops of the Eau Medicinale, injected into the jugular vein of a dog, produced rather more violent effects, than in my experiment with the same quantity of vinous infusion of Colchicum; the animal died in 6 hours, and after death the appearances of inflammation in the bowels, were more violent, approaching to mortification.

Mr. GATCOMBE having found so exact a similarity in the effects of the two medicines, in these trials, I requested him to make the following comparative experiment on the effects produced upon the stomach and bowels by the Eau Medicinale, in which there is a deposit, and the vinous infusion of Colchicum, in which there is none.

Exp. 4. One hundred and sixty drops of the Eau Medicinale, taken by the mouth, produced the same effects, and left the same appearances after death, as when that quantity was injected into the vein, only the animal lived 9 instead of 6 hours.

One hundred and sixty drops of the vinous infusion of Colchicum, were given to a puppy of the same litter; they produced vomiting, purging, and a great flow of urine; but the animal very soon recovered.

Two hundred drops of the same infusion, after an interval of several days, were given to the same dog, and the effects were the same; the dog had become much improved in his looks and condition.

Three hundred drops, after an interval of several days, were given to the same dog: effects, corresponding with

those of 160 drops of the Eau Medicinale, were produced. The dog died in 9 hours, and the appearances of inflammation after death were of the same kind, but not nearly so extensive.

From these experiments the Eau Medicinale with the deposit, produces double the irritation on the coats of the stomach and intestines, that is brought on by the vinous infusion of Colchicum: this probably arises from the local inflammation brought on by the deposit, upon the internal membrane of these viscera.

To determine as nearly as possible the effects of the deposit, when applied in a solid form, to the coats of the stomach and intestines, the following experiment was made.

Exp. 5. Six grains of the deposit of the vinous infusion of Colchicum, were given to a dog in bread and milk; in 3 hours it produced vomiting and purging, which lasted 24 hours; during the latter part of that time, there was blood in the stools, as well as in what was brought up from the stomach.

I wished to repeat this experiment with the deposit from the Eau Medicinale, but found in bottles that had been kept 7 years, the wine had become vapid, and, in this decomposed state, the acrid part of the deposit had been taken up again; so that in 12 bottles, containing different quantities, only 5 grains could be procured, which was quite inert.

Being at a loss to know whether the extractive matter deposited from the infusion, is in reality more acrid to the stomach than that suspended in it, or the circumstance of its being applied in a solid form renders it so, I requested Professor BRANDE to acquaint me, if it could be the effect of any chemical decomposition having taken place.

He favoured me with the following explanation, which is highly satisfactory. "There are certain vegetable bodies which, when infused in water, or diluted spirit, furnish a solution which lets fall a sediment, in which their activity, as purgative medicines, chiefly resides; this is remarkably the case with the wild cucumber or elaterium. The sediment is a very drastic purge; the part that remains dissolved is comparatively mild in its operation upon the bowels." This explanation of Professor BRANDE applies to the Colchicum, and we are now enabled to separate the purgative qualities of the vinous infusion of Colchicum and Eau Medicinale, from those which prove a specific for the gout, in the simplest possible manner, by keeping them in large bottles, instead of small ones, and not going too near the bottom.

It also explains what is asserted by PROSPER ALPINUS,* that the Egyptian women eat the fresh bulbs, that they may grow fat; an effect which was found to take place in the dog, while the dose was confined within such limits as not to act too violently upon the bowels.

The bulbs of the Egyptian Colchicum, when long kept, weigh one dram each; on being steeped in water they double their weight; so that the quantity of extractive matter contained in two or three recent bulbs, while combined with the mucilaginous matter, of which the bulbs are principally composed, is not likely to be sufficient to do more, than act as a brisk purgative, the occasional use of which tends to make people grow fat.

Since this Paper was read, the patient who is mentioned as having had the gout in January, has had another attack:

* Hist. Nat. Egypt. pars. 1. lib. 3. cap. 14.

it came on the 10th of July, and was removed in the same manner as the former, by the same dose of the medicine. The President of the Society also, convinced by the evidence contained in this and the former Paper, that the Vinum Colchici, in which there is no deposit, must be a less hurtful medicine than the Eau Medicinale, thought it a duty to himself and the public to make trial of it, and on the 20th of July, when the gout in his left hand and the whole of the joints of that side of the body was very severe, allowed me to give him 90 drops of the Vinum Colchici, and found that the symptoms of gout were sooner and more completely removed than they ever had been by the Eau Medicinale, of which he has an experience of seven years, having taken it regularly ever since the 17th of February, 1810, and during that time kept a regular account of the doses, their effects, and the intervals between them.

XX. *Upon the extent of the expansion and contraction of timber in different directions relative to the position of the medulla of the tree.* By Thomas Andrew Knight, *Esq. F. R. S. In a Letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. G. C. B. P. R. S.*

Read May 8, 1817.

My Dear Sir,

Many attempts have been made by writers on vegetable physiology, to account for the force with which the sap of trees has been proved by Hale to ascend during the spring, without any hypothesis having been offered, which has been thought satisfactory: and almost all, which have been offered, have been justly rejected as wholly inadequate. I have suggested in the Philosophical Transactions of 1801, 2d Part, page 333, the expansion and contraction of those cellular processes, which proceed from the bark to the medulla, which I have there called the true, or silver grain of the wood; and which have, generally, though most improperly, been called medullary processes. I have there shown, that this substance expands and contracts very considerably under changes of temperature and moisture; and I have stated that a board of oak, which has been formed by cutting across the supposed medullary processes, can scarcely be made, by any means, to retain the same form and position when subjected to various degrees of heat and moisture. I had not at that time ascertained, with accuracy, the comparative expansion

and contraction of timber, when divided in different directions relative to the medulla of the tree, and I was not in possession of any fact which enabled me to prove the existence of any such power, in a state of action, in the living tree. But experiments, which I have made at different subsequent periods, have afforded very satisfactory evidence of the presence of this power in a state of action in living trees, and have also enabled me to ascertain some facts, which appear interesting, and likely to prove useful in directing the proper mode of application of wood for various purposes, in which it is important that it should permanently retain its primary extent and form. These experiments were made upon timber of many different kinds; but as the results were all very nearly the same, I shall confine myself to those made upon the oak, the ash, the beech, and poplar.

Some thin boards of the wood of two of the above mentioned species of trees, the ash and the beech, were cut in opposite directions relative to their medulla, so that the convergent cellular processes crossed the centre of the surfaces of some of them at right angles, and lay parallel with the surfaces of others; by which means I became enabled to mark the comparative extent of their expansion and contraction when they were subjected to various degrees of heat and moisture. Both were placed under perfectly similar circumstances in a warm room, where those, which had been formed by cutting across the convergent cellular processes, soon changed their form very considerably, the one side becoming hollow, and the other raised; and in drying these contracted nearly fourteen per cent. relative to their breadth. The others retained, with very little variation, their primary form, and did not contract more than

three and a half per cent. in drying. Both were, subsequently, several times subjected to various degrees of temperature and moisture, and each expanded nearly in the same degree that it had contracted, the form of the one remaining very nearly permanent, and that of the other constantly changing.

A beech and an ash tree, each somewhat exceeding twenty inches in diameter, were felled in the end of January (at which time the buds of both had become sensibly enlarged) and a transverse section of about an inch in thickness, and necessarily of a circular form, was immediately cut off from the trunk of each, near its base. An incision was then attempted to be made with a saw from the bark to the medulla, directly in the line of the convergent cellular processes, with the expectation that these, on each side, would expand, and impede the action of the saw. The result was just what I had anticipated, and long before the saw approached near the medulla, it became so strongly compressed that my assistant could scarcely move it. A much thinner saw, which I had in readiness, was then employed; and the incision, which was kept open by a wedge, was extended to the medulla. The wedge was then withdrawn, and the opposite sides of the division instantly came in contact with great force. A second incision, similar to the preceding, was then made to commence at the bark, about an inch distant from the preceding, and to terminate, like that, at the medulla; by which means, a wedge of wood, an inch square at the bark, and ending in an edge at the medulla, and ten inches in length, was wholly detached. This, nevertheless, did not quit its position, being retained in it by the expansion of the wood from which it had been separated.

The opposite sides of the same transverse sections of wood were divided by the saw in a direction diametrically opposite to that above mentioned; under which circumstances, the expansion of the convergent cellular processes could not, as in the preceding cases, occasion any pressure upon the sides of the saw, which consequently continued to move with perfect freedom.

These circumstances led me to infer, that the medullary canal must be subject to considerable variations of diameter, with the increase or diminution of the quantity of moisture in the wood; and I conceived, that I should easily be able to ascertain the truth or falsehood of this conjecture by the following means. I selected, in winter, some parts of the stems of young trees as soon as they were felled, which I retained in such a situation as might occasion them to lose a considerable part of the water they contained, though not to such an extent as to destroy, or endanger, life. The medulla of these was then removed; and the space it had occupied was filled with cylindrical pieces of metal, which were so large that they could not be introduced without considerable force. The pieces of wood were then deposited in a damp soil, from which they absorbed much moisture; and at the distance of ten days, I found the medullary canal so much enlarged, that the pieces of metal dropped through without any pressure being applied.

I am prepared to prove, in a future communication, that the quantity of moisture in the alburnum is subject to great variations in the living tree, and therefore I conclude, that the medullary canal frequently changes the extent of its diameter.

It appears probable that, by means of this kind of expansion, the internal parts of timber trees so frequently become rifted or cleft. Winds have been assumed by some, and frost by others, as the cause of these injuries. But winds cannot possibly be the cause, as pollared oak trees, upon which these can exert but very little power, are almost always rifted; and the frost of this climate is rarely, or never, sufficiently intense to congeal the winter sap of trees. This agent must also, I conceive, act suddenly, if it act at all, and the trunks of large oaks can not suddenly be cleft asunder in silence. The oak timber of England is also much more frequently rifted than that of the north of Europe. The force with which the cellular substance of timber expands, is fully equal to produce the preceding effects. I have often seen it overcome the pressure of many tons: it is therefore greatly more than equal to give the impulse to the sap, which was observed by HALE; and as it is obviously in action in the living tree, I must retain the opinion which I formerly gave, that it is the agent by which motion is given to the ascending fluid. How it immediately acts upon the passages through which that fluid ascends, and whether that fluid passes through the cells themselves, or through the intercellular passages described in the elaborate work of Dr. KIESER,* I confess myself to be wholly ignorant, and the slow motion of the fluid, the excessive minuteness of the passages, and the varieties of directions in which it is often moving at one and the same time, will ever render this a question of extremely difficult solution.

There is another kind of contraction in timber whilst drying, and of expansion when subsequently wetted or moistened,

* Mémoire sur l'Organization des Plantes.

which is observable only in lifeless wood; and which has apparently no connection with the power by which the sap is raised in the living tree. The interior and older layers of wood are much more solid and specifically heavy, than the external layers in the same tree; and the latter, consequently, contract more longitudinally in drying than the former, and the edge of every board (that has been cut with surfaces nearly parallel with the line of the convergent cellular processes) which lay nearest the medulla in the tree, will therefore in drying become convex, whilst the opposite edge will become concave. The ill effects of this are often felt when oak timber is employed to form joists, part of these in drying always rising above, and others sinking below the first and proper position. The cause of some musical and other instruments being put out of order by changes of weather, whilst others, apparently similarly constructed, are free from such defects, may probably be traced to one of the sources abovementioned.

I am, my dear Sir, &c.

T. A. KNIGHT.

Downton, April 26, 1817.

The Right Hon. Sir Joseph Banks, Bart. G. C. B. P. R. S.

XXI. *Observations on the temperature of the ocean and atmosphere, and on the density of sea-water, made during a voyage to Ceylon. In a Letter to Sir* Humphry Davy, *LL. D. F. R. S. By* John Davy, *M. D. F. R. S.*

Read May 22, 1817.

My Dear Brother,

According to the promise contained in a former letter, I proceed to give you a short account of the observations which I made during my late voyage from England to Ceylon. At present, I shall confine myself chiefly to three topics, the specific gravity of the water of the ocean, and its temperature, and the temperature of the atmosphere; subjects of some importance in the natural history of our globe; and in which, I know, you are interested. Incidentally I shall notice the height of the barometer, the direction of the winds, and the state of the weather.

For the sake of brevity, I shall present the principal results of my observations in the form of a table, to which I shall add some explanatory notes and general remarks.

Time.	Latitude by Observation	Longitude by Chronometer.	Specific gravity of sea water at temp. 60.	Maximum temp. of the air in the 24 hours.	Minimum.	Mean.	Maximum temp. of the sea in the 24 hours.	Minimum.	Mean.	Barometer.	Winds.	Weather, &c.
	° ′	° ′		° ′	° ′	° ′	° ″	° ′	° ′	° ″		
Feb. 12	N. 9.	W. 0,30				40,			48,		South	Clear.
14	48,28	10,20	10251	51,	49.	50,			51,	29,	SW	Cloudy with occasional sunshine.
15	47·53	10,28	10264	52,	50.	50, 7			52,	30,1	WNW	
16	46,28	11,50	10256	50, 5	49.	50,			52,	30,	North	
17	44·32	14.10		50,	49.	49. 6			52,	30,	NE	
18	42,54	15,47	10256	53.	51.	52,			53.	30,	ENE	
19	40,48	16,34	10256	53.	52,	52,25			53.	30,	SE	
20	40,21			56,	53.	54, 3			55,			Clear. A complete calm.
21	40,12			58,	56,	56, 6			59,		SSW	Cloudy, wind strong, sea rough.
24	39,24			57.	55.	56,			58,	30,4	NE	Clear, wind moderate after a strong gale for two days from the SW.
25	37, 5	21.35	10256	59,	58,	58, 5			59,	30,5	SE by E	
26	34,54	22, 5	10270	60,	59.	59. 5			60,	30,1	SE	
27	33.14	21,59	10264	61,	60,	60,			63,	30,2		
28	31,58	22.24	10260	61,	6c.	60,			63,	30,		
29	30,41	22,23		61,	58,	59. 5			64,	30.1		
Mar. 1	28,25	23, 5	10266	64,	61,	63,			65, 5	30,2	ESE	
2	26,36	23,15	10273	64. 5	63,	63. 8			66,	30,2	E by S	
3	23,30	23, 6		65,	63. 5	64. 5	68,	66,	67,	30,2		Hazy.
4	21,32	22,36		66, 5	65.	65, 8	68,	67.	67. 5	30,1		Cloudy.
5	19. 1	22.47	10256	66, 5	65,	65, 8			68,	30,1	ESE	
6	16,47	22,37	10267	67,	63,	65. 3			68, 5	30,2	NE by E	Fine.
7	15. 4	21,26	10276	68,75	67.	64. 6			71,	30,		
8	12,56	20,20	10275	69, 5	68, 5	68, 8			71, 5	29,9	ESE	
9	11, 8	19,		73,	71, 5	72. 3			74.75		NE by E, E by S	
10	9,42	19,20	10276	76,	73	74. 3			76,	29.9	E by S	
11	8,23	19,16	10277	76.	71,75	74,			77,	29.0	N by W to S by E	
12	6,57	19,10	10277	79. 5	77.	77. 3	79,25	77. 5	78, 6	29.9	NW	
13	5,50	18,41		80,	77.25	78. 6	80,75	79. 5	8 , 4	29,8		Cloudy.
14	5. 4	18,50		79. 5	74.	76,	80, 5	78, 5	80,	29,	North	Thunder storms with heavy rain.
15	4. 9	19,15	10277	80,	78, 5	79.	81, 5	80,	80, 7	29,	S, SE by N	Pleasant.
16	4, 2	18,44	10275	82,	78, 5	79. 6	83, 5	80,	81, 8	29,4	SE by E	Calms and squalls with thunder storms and heavy rain in succession.
17	4. 0	18,30	10270	80,	74,	77.	81,	80, 5	80, 5	29.		
18	2,58	18,44	10270	77.75	76, 5	77. 5	79,75	79. 5	79. 5	29.	SE to E	
19	2,27	19, 2		80,	77.	78, 6	7?,75	79.	79. 5	29.	SSE	Fine.
20	1,20	21,10	10264	80,	78,	79.	79.	79.	79.	29.	ESE	
21	S. 0,12	21,50	10264	79. 5	78,	78, 8	78.75	78,	78, 5	30,2	ESE	
22	1,28	22,20	10264	79. 5	78.	79.	79. 5	78. 5	79. 2	30,	SE by S	
23	2,29	23. 5		79.75	78.75	79.	79, 5	79.	79.25	30,	SSE	
24	4,13	23.15		80, 5	78.25	79. 3	80,	79.	79.75	30,	ES by S	
25	6,27	24.13		80, 5	79.	79, 6	80, 5	79.75	80,	30,		
26	8,46	24,21		80, 5	79.	79. 3	80,	79.	79.75	30,3	SE by E	
27	10,30	24.25	10263	80,	79.	79. 3	80,	79.75	79.75			
28	12,12	24,50		79,25	78,	78,	80,25	79. 5	80,	30,2	ESE	Cloudy, showers.
29	13.45	25. 7		80, 5	77,	79,	80, 5	80,	80,25	30,2	SE by E	
30	15,35	26,		81,	78.	9.	80,75	80,	80. 5	30,2	ESE	
31	17.42	27,		80,25	77,25	79.	80,75	77. 5	80,	30,2	SE by S, ENE	
Apr. 1	19.47	27,23		81,	78,	79. 4	81,	79. 5	80,	30,1	E by S	Very fine.
2	21, 3	27,27		80,	77. 5	78, 5	80.75	78. 5	80,	30,2		
3	22,36	26.30	10264	80,	77. 5	77. 7	80,25	79.	79.75	30,2	ENE	
4	23,44	26,29		80,	76.	77. 5	80, 5	79.	79.75	30,1		
5	24,22	26.27		79.75	76.	77.	80, 5	77.75	79.			

Time.	Latitude by Observation	Longitude by Chronometer.	Specific gravity of sea water at 60° Fahr.	Maximum temp. of the air in the 24 hours.	Minimum.	Mean.	Maximum temp. of the sea in the 24 hours.	Minimum.	Mean	Barometer.	Winds.	Weather, &c.
	° ′	° ′		° ′	° ′	° ′	° ′	° ′	° ′	° ′		
Apr. 6	S. 25,22	W26,33		80,	75, 5	77, 8	79, 5	77, 5	78, 2		NNE	
7	26,30	26,29		79,	73,	75,75	78, 5	76,75	77, 3	30, 5	NW by W, ESE	Unsettled, frequent showers, lightni:g.
8				73, 5	70,	72, 6	78,	75, 5	76, 8	30, 7	ESE to S	Tempestuous, sun obscured, heavy rain, a great swell from the SE.
9				73,	70,	71, 6	77,	72,	74,	31,	ESE	
10	27,50			71, 5	70,	70, 7	73,	72,	72, 4	30, 5		
11				71,	68,	69, 5	72, 5	71,75	72,	30, 3		
12	29,17	25.25		73,	70,	71, 3	72,	70,	71,	29, 8	NE by E	Pleasant. [night fine.
13	30,16	23,10		71,75	70,	71,	71, 5	69, 5	71,		N by E, NW	Variable, the day squally, the
14	30,20	23, 5		74,	70,	71, 6	74,	71,	72,			Calm.
15	30,25	20,	10256	73, 5	70, 5	71, 7	72, 5	71,	71, 7	30, 0	NE to NW	Pleasant.
16	31, 6	18,55		71,	66,	66, 5	71,25	67,	68, 6	29, 9	WSW, SSW	
17	31.48	17, 1		66,75	64,	65, 5	69,25	65,	68,	30, 7	SW by W	
18	32,12	16, 3		67,75	66,	67, 7	70,	68,	69,	30, 4	NW	Extremely damp, at night a little dew fell, the first observed.
19	33,29	15.17		71,	66,	68, 3	68, 5	65,	68,	30, 2	NNE	
20	33,58	10,48		67, 5	65,	66,	65.75	63,	65,	30, 1		
21	34.29	7,56		68,	63,	66,	66,25	65,	65, 7	30,		
22	34,26	5,30		64, 5	60,	62,	63,75	62, 5	63,	30, 1	SW	Air dryer, no dew.
23	34,25	3, 2	10253	64,	60,	62, 2	65,	63,75	64, 5	29, 7	W	Cloudy, some rain.
24	33,25	2,50		65,	61,	63, 5	66,	65,	65.25	29, 9	W by S	Cloudy.
25	33,43	0,43.		63, 5	60, 5	61, 6	65,	63, 5	64, 2	30,	SW, SSE	
26	33,28	1, 2		62, 5	59.75	61,	64, 5	62,75	63, 6	30,	SW	
27	34,14	1, 1		61,	59,75	60, 3	64, 5	62, 5	63,75	30, 2		
28				63,	59, 5	62, 2	64, 5	62,	62, 6		E by N	Hazy.
29	35,93	2,13		64, 5	61,	62, 6	64,	62,	63,		W by N	Cloudy.
30	34,45	5,31	10251	65, 5	62, 5	63,75	64, 5	62, 5	63, 4			
May 1	34,36	7,13		66,	63,	64, 4	65,	61,75	64,	30, 1	NW	Fine.
2	34,32			64,	61,	64, 6	66, 6	63, 5	65, 2	30, 2	NW, SSE	
3	33,26	10,11		61,	59,	60, 5	63, 5	60,	62,	30, 3	SE by S	
4	34,32			60,	58,	58, 5	63,	61,	61, 9	30, 4		Cloudy.
5	35,22	11,23		60,	58,	58, 4	65,	61,75	62, 7	30, 5	SSE, SE by S	Fine.
6	34,28	12, 7		59,	58,	58, 9	65,	59.25	63, 4	30, 5	ESE, E by N	
7	35, 1	13,20		61, 5	58,	60,25	64,	60,	61, 6	30,		Cloudy.
8	34,23	14.17		63,	57, 5	60,25	64, 5	61,	63,	30, 3	NW, SW	Squally.
9	34, 1	15,31		58,	56, 5	57,	63, 5	62,	63,		SW by S	
10	34,24	16,45		62,	59,	60,70	62,75	61, 5	62,	30,7 to 29	NW, SSW	More clear and moderate.
11	34, 1	17,51	10259	60,	57,	58,	62, 5	57,	60,	30, 2	SWS	
12				58,	54,	56,	57,	55,	56, 2	30, 2		In sight of Table Mountain, and within soundings.
June 3			10251	56,	51,	54, 1	55.25	53,	55,			All day in soundings.
4				60,	55,	58, 4	62,	57,	60, 3		SSW	Still in sight of land, water still greenish,
5				62,	59,	59,75	65, 5	58,	63,		SSW	Out of sight of land, water blue.
6	34,15			62,	59,	60, 4	63, 5	62,75	63,		ENE	Pleasant.
7	34,53	15, 8		63, 5	60,	61,25	64,	61,	62, 6		NNE	
8	35, 1	16,42		61,	60,	60, 5	61, 5	59,75	60, 8			Cloudy.
9	36, 4	19,26		64,	59, 5	61, 6	66,	60, 5	60, 3		NW	Tempestuous.
10	35,57	24,	10253	62,75	58,	59, 1	71, 5	61,	66, 9		NW, SW	
11	35,36	27,19		58, 5	57,	57, 6	67, 5	61,	65,		SW	
12	35,50	28,30		64,	57,	62,	67,	64,	66, 3		NW	Improving.
13	35,53	30,53		64,	61,	62, 9	65,	63,	64, 2			Pleasant.
14	35,52	33,54		63,25	62,	62, 6	65,	63, 5	64, 4		SW, N by W	Fine.

Time.	Latitude by Observation	Longitude by Chronometer.	Specific gravity of sea water at temp. 56.	Maximum temp. of the air in the 24 hours.	Minimum.	Mean.	Maximum temp. of the sea in the 24 hours.	Minimum.	Mean.	Barometer.	Winds.	Weather, &c.
	° ′	° ′		° ′	° ′	° ′	° ′	° ′	° ′	′		
June 15	S. 35.31	W 36. ′		65. ′	59. ′	61. 3	64. 5	63. 5	63. 9	30.3	W by N, SW	
16	24.23	37.56		62.	60,75	61. 5	65. 5	64.	65.	29,8	SE, NW	Cloudy.
17	34.44	4[illegible]. 7		65.	58.	61. 6	65.	61.	64.	30,3	NW, SE by S	A thunder storm.
18	33.48	42. 3		60.	58. 5	59. 5	65.	62.	63. 4	30,6	E by S	Pleasant.
19	34.48	42.24		63.	59. 5	61.	64.	61. 5	63.	30 to 31	NE	Cloudy, tempestuous, showery.
20	34.54	45.17		63.	61.	62.	64.	62.	62. 7	30,1	N	Incessant rain and thunder and lightning.
21	35. 7	48.40		63.	58.	61.	62.	60.	60. 9	30.	WNW	Improving.
22	31.34	51.35		62.	57.	59. 8	62.	60.	60. 9	30,1	W	
23	33.30	55.16		60.	57.	58. 7	62,25	61.	61. 5	30,1	W by S	Moderate.
24	32. 4	58.18	10260	60.	57.25	58. 3	62.25	61. 5	62.	30,1	SW, SE	Pleasant.
25	31. 2	60.		59.	56. 5	57. 3	62.	60.75	61. 2	30,1	NNW	Gloomy.
26	30,39	60.33		59.	57.	57. 2	63.	61.	61. 8	30,3	E by S, ENE	
27	30,48	60,36		61. 5	59. 5	60. 6	61. 5	60,25	61. 3	30,3	NE	
28	31.23	63.17		63.	61. 5	62. 5	63.75	62.	62. 5	30,2	N by E	
29	31,20	64.34		63. 5	62.	63.	63,75	61.	62. 5	30,1		
30	30.53	65.38		67.	62. 5	63.	65. 5	63. 5	64. 5	30,4	N, W by N	Fair.
July 1	30,58	65.45		66.	61. 5	63. 6	65.	63.25	63. 6	30.2	SW	Gloomy, some rain.
2	29.23	66,17		62.	62.	60. 7	67.	64.	65. 5	30,3	SE	Fair.
3	27,18	66,30		64.	60.	62. 4	68,75	65.75	67. 3	30,4	E	
4	24.45	65.45		67.	64. 5	65. 8	70 75	68.	69. 4	30,3	E by S, E by N	
5	21,45	65,25	10259	69.	67.	68.	72.	70.	71.	30,4	SE	
6	19.53	63. 1		71.	68.	69.	74.	72.	72. 5	30.		Off Rodriguez.
7	19.44	62,50		71.	69.	69. 5	73. 5	71. 5	72. 3	30,2	S by W	At night in sight of the Isle of France.
17	19.15	57.11		72,75	71.	71. 6	74. 5	72.	73.	30,1	E by S	Fair, out of sight of land.
18	18,14	57.30		74. 0	72.	72. 3	74.	72. 5	74. 7	30,2	E	
19	17. 6	57. 3		74. 5	72.	73. 4	75.	72. 5	74.	30,2	E by S	
2[illegible]	15.46	58.		74. 5	72. 5	73. 6	76,25	75.	75. 8	30,2	ESE	Showers.
21	14.22	58.48		75.	74. 5	74. 7	76. 5	75.	76.	30,2	E	Night squally.
22	12,17	59. 3		76. 5	73. 5	75. 1	77.	74.25	75. 6	30,2	E by S	Fair.
23	10.12	60.		77.	74.75	75. 1	77. 5	75.	76,25	30,		
24	7.54	60,23		77. 5	72.	74. 6	77. 5	75.	75,75	30,		—— night squally.
25	5.42	60,51		76.	75.	75. 4	77.	75. 5	76. 2	30,1	SE	Squalls with rain.
26	4. 6	61.45		78.	76.	77.	78,25	75.75	76. 5			
27	2.40	62.51		78.	76,25	77.	79. 5	78. 5	79.		E by N	Cloudy, squalls with rain.
28	1,40	63,35	10253	79. 5	77. 5	78.	81.	77.	79. 1		NE	Improving.
29	1,14	63,30		79.	75. 5	78.	80. 5	78.	79. 4			Rainy night.
30	0,19	65.11		78,25	77.	77. 8	81.	79.	80. 2		SE	Pleasant.
31	N 0,22	66,36		81.	73. 5	77. 5	82. 5	78.	80. 5		W by N	Day calm, night rainy.
Aug. 1	0,30	67.52		77,75	75.	77.	80. 5	79. 5	80.		S by W	Much rain.
2	0,26	68,58		78.	76.	77. 5	81.	80.	80. 4		S by E, S by W	Unsettled, some rain.
3	0,58	70, 6		81.	78. 5	79. 9	81. 5	80.	80. 7		SSW	Fine
4	1,27	71,51		80. 5	78,75	79. 9	81,75	80.	80. 7		S by W	
5	1,17	73,53		81.	79.	79. 5	81. 5	79,75	81.		S	
6	1,37	75,36		80. 5	78.	79. 9	81.75	79. 5	80. 6		SW	
7	2,10	76,37		82.	76.	78. 8	83. 5	80.	81. 7		SW by W	Part of the day calm.
8	2,34	77.25		81.	77. 5	79. 6	82,25	79,25	80. 7		N by W	Pleasant.
9	2,30	77,53		82.	75. 5	77.	84.	78. 5	80. 3			Calm.
10	3,22	79. 4		79.75	77.	77. 6	79.75	78,25	79.			Calm till 4 P. M.
11	5,17	79.42		79.75	76,25	77. 6	79.	76.	77. 5		NW	Frequent squalls.
12	6,24			77.75	75.75	76. 6	78.	76.	76. 9		WNW	Pleasant, in sight of Ceylon, in soundings.

In all the experiments on the density of sea-water, the results of which are recorded in the Journal, the water used was taken from the surface of the ocean, in a large clean bucket. The results introduced before we passed the equator the first time, were procured at sea; the remainder, from 0° 12″ south lat. to Ceylon, were obtained on land from experiments made on specimens of water preserved in well corked phials. In the experiments on board ship, as soon as the water was drawn, its temperature was ascertained, and then it was immediately weighed. The balance employed was not very delicate, for a very delicate instrument does not answer at sea, on account of the ship's motion; however, it was pretty readily affected by $\frac{1}{10}$ of a grain. The glass vessel in which the water was weighed, was such a one as is commonly used at home; its capacity was equal to about 300 grains. In the experiments on shore, the same vessel was used, but a different balance, one of a more delicate construction. I have chosen the temperature 8c° FAHRENHEIT, for which I have calculated all the results, because it is nearly the mean annual temperature of this place, and nearly the mean at sea, in the intertropical regions.

The experiments made at sea I do not of course value so much, as those made on land: considered, however, merely as approximations to the truth, which I am sure they are, the results favour the general conclusion already formed by some philosophers, that the ocean resembles the atmosphere in being (cæteris paribus) of nearly the same specific gravity throughout.

And farther, they lead to the conclusion, that the slight

variations of specific gravity observed, do not regularly conform to the difference of temperature.

That the specific gravity of the water of the ocean, in all its parts, however remote, should be nearly the same, is easily explained; it is indeed what might be expected from theory. It is more difficult, it appears to me, to account for the slight variations; I may remark, they appeared to me greatest when the sea was rough and agitated; and once the specific gravity of the water seemed diminished by a heavy fall of rain, viz. in lat. 4° north, and in long. 18° 13″ west, where we experienced a quick succession of tropical squalls.

Whether there is a specific gravity peculiar to the water of each zone, as a modern traveller of high authority endeavours to prove, I am greatly in doubt. From my own experiments, in which I cannot but put some reliance, I feel much inclined to infer the contrary, and especially from those made on land, which I know to be perfectly accurate. Several of these agree in giving the same specific gravity to specimens of water taken from parts of the ocean very remote from each other; for instance, the water from lat. 0° 12″ south, and 22° 36″ south, and that from 34° 25″ south, and the water that washes the shores of Colombo.

For ascertaining the temperature of the air and of the water of the ocean, I used delicate pocket-thermometers, the bulbs of which projected about an inch from the ivory scale. In the experiments on the temperature of the ocean, the water was tried the instant it was drawn, before it was affected by the air. To find the temperature of the air, I always chose the coolest part of the ship on deck, and always put the in-

strument in the shade, and exposed it to the wind, taking care not to bring it near any surface that had the power of radiating much heat, circumstances, I need not remark, of importance to be attended to, and, in consequence of the neglect of which, the temperature at sea, in the intertropical regions, has by most observers been overrated.

During the greater part of the voyage, observations were made every two hours, on the temperature both of the air and of the water; and with the kind assistance of the mates of the ship, Messrs. SLEIGHT and POWELL, intelligent and obliging men, they were carried on during the night as well as the day.

I am not aware that the law of the diurnal variation of the temperature of the atmosphere at sea, has been described by any writer. From the numerous observations, which I had an opportunity of making, between and bordering on the tropics, it appeared to me perfectly regular at a great distance from land, when the weather was fine, and the wind steady. In these circumstances, I found the air at its maximum temperature precisely at noon, and at its minimum towards sunrise. I shall give in illustration of the fact two instances from my note book.

April 2d. S. lat. 21° 3″. W. long. 27° 27″. Wind E. by S.

Hour.	Temperature.
6 A. M.	78
8	79
10	79,5
12	80
2 P. M.	79,25
4	79

Hour.	Temperature.
6	78°,5″
8	78
10	78
12	77,75
2 A. M.	77,75
4	77, 5
6	77, 5

April 5th. S. lat. 24° 22″ W. long. 26° 27″. Wind ENE.

Hour.	Temperature.
6 A. M.	76° ″
8	77,5
10	78,25
12	79,75
2 P. M.	78, 5
4	77,75
6	77, 5
8	77
10	77
12	76, 5
2 A. M.	76, 5
4	76

Here we perceive the variation of the temperature of the air, following the course of the sun, pretty considerable whilst it is above the horizon, and very insignificant during the night; and this, I may remark, is a general fact at sea, and one of the principal features of difference between the temperature of the atmosphere over the land, and over the ocean.

The law of the regular variation of temperature, is frequently interrupted. Even in fine weather, when the air is not in motion, it is subject to interruption. During a calm, the variation of temperature is nearly the same as on land, the maximum degree of heat not being at noon precisely, but some time after, and for the same reason; because there is an accumulation of heat, and not only in the ship, but actually in the water itself, as I may show by noticing the temperature of the air and of the sea, during even a short calm, hardly of 24 hours duration.

August 7th. N. lat. 2° 10″ E. long. 76° 37″.

Hour.	Temp. of air.	Of the sea.
6 A. M.	78°,5	80°
8	79,5	81
10	80,5	81,5
12	82	82,5
2 P. M.	82,5	83,5
3	82	83,5
4	81,5	

But the law is more remarkably interrupted during storms and unsettled weather, as a couple of instances will be sufficient to prove.

March 17th. N. lat. 4°. W. long. 18° 30″

Hour.	Weather.	Temp. of air.	of the rain water.
8 A. M.	Clear	80	
11	Rain approaching	77	76
11° 30	Just passed	74	78
12	Cloudy	79	
1 P. M.	After a shower	76,5	76
4		75	74

March 27th. S. lat. 10° 30". W. long. 24° 25".

Hour.	Weather.	Temp. of air.
5 A. M.	Fair	79
6	Rain approaching	78
6 30	Raining heavily	75,5
7	Rain just ceased	76,5
8	Sunshine	79,25
9	Raining	76
10	Cloudy	79,5
12	Fair	80,5

The showers in each instance were accompanied by hard gusts of wind, and thunder and lightning. The rain-water, the temperature of which was ascertained, was collected in a glass as it ran from the awning.

The equatorial regions appear to be particularly subject to storms, violent rain, and electrical phenomena, the effect of which, in diminishing the temperature, seems to afford a natural explanation of the comparative coolness, both of the atmosphere and the ocean, that we experienced each time we passed the line.

The temperature of the sea, it has been asserted by some writers, is subject to little or no diurnal variation. That this remark is far from correct, is evident from the slightest inspection of the Meteorological Journal; it is an opinion that could be formed only from hypothetical views, ill founded. The fact, as the Journal exhibits, is, that the diurnal change of the temperature of the sea is very nearly as great as that of the incumbent atmosphere. From all the observations I could make, when the circumstances were most favourable to accurate results, when the weather was fine, the sea smooth, and the land at a great distance, it appeared to me,

that the maximum temperature is about three in the afternoon, and its minimum towards sunrise. I shall give a single example in detail.

April 5th. S. 24° 22″. W. long. 27° 8″.

Hour.	Temp. of the sea.
8 A. M.	79, 25
10	79, 5
12	79, 5
2 P. M.	80
4	80, 5
6	80
8	79, 5
10	79
12	78, 5
2 A. M.	78
4	77,75
6	76

Like the atmosphere, the ocean is subject to irregularities of temperature. This fact is proved by the Journal in an ample manner. The causes which produce these irregularities may be divided, very generally, into three kinds, tempestuous weather, shoals, and currents.

Independent of other modes of operation, and they are various in tempestuous weather, superficial currents appear to be established in the course of the prevailing winds. If the wind be from a cold quarter, the temperature of this current is comparatively low, and *vice versâ*. This fact is manifest in the effect of the gales we experienced between the 7th and 12th of April, during which time, being south of the

equator, and the wind blowing from the south, the temperature of the sea was considerably reduced.

Where the sea is shallow, it is now a well established fact,* that the temperature of the water is comparatively low; an important circumstance, highly deserving the attention of the practical navigator; it may forewarn him of a bank in the darkness of night, when nothing else would indicate it, and put him on his guard when approaching low shores and shallows, time enough to avoid their dangers. In advancing towards the Cape of Good Hope, and in doubling that promontory, and in making Ceylon, I collected some observations on this subject, the results of which I shall now introduce. On making Table-bay, before land was to be seen, there was a decided fall of the temperature of the water, viz. from above 60 to 58, thus,

May 11th. S. lat 34° 1″. E. long. 17° 51″ at

8	A. M. the temp. of the water was	62,5
10	- - - -	62,5
12	- - - -	61,5
2	P. M. - - - -	61
5	- - - -	60
10	- - - -	58
12	- - - -	58
2	A. M. - - - -	58,5
4	Land in sight - - -	59
7	About 20 miles from land -	58
8	- - - -	57

* Observed by Dr. FRANKLIN, Mr. J. WILLIAMS, &c. See WILLIAMS's Thermometrical Navigation. Philadelphia 1790

10	- - - - -	56
12	- - - - -	56
2 P. M.	- - - -	55
4	- - - -	56
8 In soundings	- - - -	56,5
10	- - - -	56,5
12	- - - -	55
4 A. M.	- - - -	55
6	- - - -	56,5
8	- - - -	56,5

During these two days we were gradually approaching land, at the average rate of about two miles an hour. The observations were continued, till we were within about two miles of the shore. The observations I made on leaving the bay, corresponded with the foregoing, as nearly as could be expected, considering the track was not precisely the same, and the cold season more advanced.

		° ′
June 3d.	8 A. M. Half a mile from land, temp. of water	58
	10 About three miles from land	54,25
	2 P. M. Off Robin Island, nine miles from Cape Town, in ten fathoms water -	55,25
	4 - - - - -	55,25
	12 - - - - -	54, 5
	2 A. M. - - - -	54, 5
	8 - - - - -	57, 5
	10 - - - -	57
	12 - - - - -	60
	2 P. M. - - - -	61
	4 - - - - -	62

Before four in the afternoon we were out of sight of the Cape of Good Hope, and in deep water.

In approaching Ceylon, and particularly the southern shore of the island, where the mean annual temperature appears to be about 80°, little or no change of temperature could be expected on entering shallow water; yet we experienced a manifest change, a reduction of at least two degrees on coming into soundings. When we were in north latitude 5° 17″, and east longitude by chronometer 79° 42″, the temperature of the water began to fall; in the morning at eight, it was 78° 5″ and at ten at night it was 76° 5″. Next morning, land was discovered.

From the observations, in general, on the temperature of the water, recorded in the Journal, there is reason to believe, that during the whole voyage we were frequently encountering currents. Many of the results stated, are scarcely to be explained on any other hypothesis. When the temperature of the water became suddenly reduced, I inferred we were either in a current from the poles, or over some high ground in the bed of the ocean; and the former conclusion was almost constantly confirmed by other observations. And on the contrary, when the temperature of the water experienced a sudden increase, I inferred that we were in a current flowing from the equatorial regions. The only current we passed, that appears to me to require particular notice, is the well known one, that flows round the bank of Lagullas, from the south-east coast of Africa. It is marked in all charts, and it has been pretty minutely, and very scientifically described, and its course explained by Major RENNELL,

but hitherto, I believe, no notice has been taken of its high temperature, or of the effect which I believe it has, in producing a curious phenomenon on the summit of the Table-mountain, not yet accounted for, viz. a dense covering of mist called the "table-cloth," which universally appears when the wind blows from the south-east. I shall copy from my notes, taken at the time, the observations I made in crossing this current.

June 10th. S. lat. 35° 57″. E. long. 24°.

Hour.	Temperature of the Sea.
6 A. M.	61
8	71, 5
10	70, 5
11	70
12	68
1 P. M.	68, 5
2	67, 5
4	68
5	67
6	66, 5
7	67
8	67
9	67
10	66,75
11	67
12	67
1 A. M.	67
2	67
3	61
4	61

Hour.	Temperature of the Sea.
5	64
6	66,75
7	66
9	67
10	67, 5
12	66
2 P. M.	67, 5
4	65, 5

Now, judging from the change of temperature, we appear to have suddenly passed from the bank of Lagullas into the current that flows round its borders. Major RENNELL, I believe, observes, that at the border of the bank, the current is strongest; the high temperature of the water there, at least ten degrees above the neighbouring sea, is readily accounted for on that idea. We appear to have continued in the current seventeen hours, the course the ship was going was nearly due east, her average rate 7.65 miles an hour, and hence, supposing we were sailing immediately across the stream, as probably we were, or very nearly, its width may be inferred to be about 130 miles; a distance little differing from that commonly assigned to it. Having traversed this current, we seem from the low temperature of the water for two hours, to have been passing a bank twelve miles wide, and then to have entered a second current running in the same direction as the first.

I have alluded to a connection between these currents and the covering of dense mist, that occasionally occurs on the Table-mountain, called the "Table-cloth." The connection is evident, and readily explained. The phenomenon only

presents itself when a cold wind blows, viz. the south-east. This wind must condense the aqueous vapour rising from the warm current, and carry it towards the land. During the short stay we made at the Cape, I once had an opportunity of seeing the mist advancing; it came rapidly over the surface of the sea, which it entirely concealed, whilst the air above was perfectly clear; it soon reached the land, spread along the coast gradually, ascended the mountain, and there remained almost stationary, enveloping the summit, sometimes encreasing and descending on the opposite side overhanging Cape Town, and sometimes diminishing and retreating. That it should remain so nearly stationary on the top of Table Hill, whilst the south-east wind continues, is not surprising, considering the height of this hill, 3582 feet above the level of the sea, its precipitous sides, and the extensive surface of its top; nor is it strange, that it should rarely descend, except when the wind blows hard, taking into account the situation of the ground beneath, sheltered and warm, and the site of a large town, from which a current of hot air must be constantly rising.

I cannot conclude, without insisting with Mr. JONATHAN WILLIAMS on the use of the thermometer at sea; if commonly employed, and the observations made with it recorded, a general knowledge might soon be obtained of the average temperature of all parts of the ocean, and a fund of curious and useful information might be collected, especially respecting currents and shoals, that to practical navigators could not fail of being highly serviceable.

In another letter, I propose communicating to you the

observations I have collected on the temperature of man and other animals in different climates. The experiments were made during my voyage, and during my stay at the Cape, and the Isle of France, and my residence at this place.

I remain, &c.

JOHN DAVY.

Colombo, Nov. 3, 1816.

XXII. *Observations on the genus Ocythoë of* Rafinesque, *with a description of a new species. By* William Elford Leach, *M. D. F. R. S.*

Read June 5, 1817.

Pliny, Aldrovandus, Lister, Rumphius, d'Argenville, Bruguière, Bosc, Cuvier, and Shaw, have described a species of this genus, that is often found in the *Argonauta argo* (common paper-nautilus) and which they have regarded as its animal, since no other inhabitant has been observed in it.

Sir Joseph Banks, and some other naturalists, have always entertained a contrary opinion, believing it to be no more than a parasitical inhabitant of the Argonaut's shell, and Rafinesque, (whose situation on the shores of the Mediterranean, has afforded him ample opportunities of studying this animal, and of observing its habits) has regarded it as a peculiar genus, allied to the *Polypus** of Aristotle, residing parasitically in the above mentioned shell.

Dr. Blainville, ten months since, when speaking of the *Argonauta*, said, "animal unknown," and he has lately informed me, that he has written a long dissertation to prove, that the *Ocythoë* of Rafinesque, does not belong to the shell in which it is found.

The observations made by the late Mr. John Cranch, zoologist to the unfortunate Congo expedition, have cleared from my mind any doubts on the subject. In the gulf of

* Sepia octopodia Linné.

Guinea, and afterwards on the voyage, he took (by means of a small net, (which was always suspended over the side of the vessel) several specimens of a new species of *Ocythoë*, which were swimming in a small *argonauta*, on the surface of the sea.

On the 13th of June, he placed two living specimens in a vessel of sea water; the animals very soon protruded their arms, and swam on and below the surface, having all the actions of the common *polypus* of our seas: by means of their suckers, they adhered firmly to any substance with which they came in contact, and when sticking to the sides of the basin, the shell might easily be withdrawn from the animals. They had the power of completely withdrawing within the shell, and of leaving it entirely. One individual quitted its shell, and lived several hours, swimming about, and showing no inclination to return into it; and others left the shells, as he was taking them up in the net. They changed colour, like other animals of the class cephalopoda; when at rest the colour was pale flesh-coloured, more or less speckled with purplish; the under parts of the arms were bluish grey; the suckers whitish.

The *Ocythoë* differs generically from the *polypus*, in having shorter arms, with pedunculated instead of simple suckers; the superior arms too are dilated into, or furnished with, a wing-like process on their interior extremities.

All the internal organs are essentially the same as in the *polypus*, although they are somewhat modified in their proportion; but as these differences may be the result of the contraction caused by the spirits, in which they are preserved, it may be more prudent not to dwell on them. Two cha-

racters, however, which I could not discover in the *polypus*, may be mentioned, namely, four oblong spots on the inside of the tube, resembling surfaces for the secretion of mucus; two inferior and lateral, and two superior, larger, and meeting anteriorly. On the rim of the sac, immediately above the branchiæ, on each side, is a small, short, fleshy tubercle, which fits into an excavation on the opposite side of the sac. This character, which, with slight modifications is common to this genus, to *loligo* and *sepia*, does not exist in the *polypus*.*

Although the superior arms are stated to perform such different functions from those of the *polypus*, yet they are supplied in the same manner, and from the same source with nerves. The muscles of these parts were in too contracted a state, to enable me to ascertain if they were in any degree different from those in the same parts of its kindred genus.

The general form of the body of this species of *ocythoë*, is the same as that of the common *polypus*, and it is covered by the same integuments, without any surface adapted either to adhere to, or to secrete, the shell in which it is found. The sexes differ as in the *polypus*.

Ocythoë Cranchii.

O. corpore purpureo-punctato, brachiis subtus cærulescente-griseis; superioribus membranâ spongeosâ pallidâ maculatâ.

The superior arms are generally attached to the side of the membranes (fig. 5. Pl. XII.); but in one specimen the membranes adhere only by their base, below the apex of the

* The rudiment of the bone, which occurs in the *polypus*, (as has been observed by Cuvier) is not to be found in the *ocythoë*.

arm, fig. 6. The membrane is subject to great variation in size and form, and is often different on the arms of the same individual.

One male only was sent home, all the others were females, which had placed their eggs in the spiral part of the shell.

One female, that had deposited all her eggs, withdrew completely within the shell, as in fig. 3; her body on one side had all the impressions of the shell, and the suckers on all the arms were diminished in size, as if from pressure.

EXPLANATION OF PLATE XII.

Fig. 1. Ocythoë CRANCHII sitting within the shell.

Fig. 2. The animal without the shell.

Fig. 3. One completely retracted within the shell.

Fig. 4. Ditto taken out of the shell, showing the impressions of the shell on the body.

Fig. 5. Left superior arm (common appearance) magnified.

Fig. 6. Right superior arm (variety) magnified.

Fig. 1.

Fig. 2.

Fig. 3.

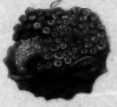


Fig. 4.

Fig. 5.

Fig. 6.

XXIII. *The distinguishing characters between the ova of the Sepia, and those of the vermes testacea, that live in water, explained.* By *Sir* Everard Home, *Bart. V. P. R. S.*

Read June 5, 1817.

Linnæus was led into an error respecting the animal that forms the shell argonauta, by the circumstance of a species of sepia having been often found in this shell. This erroneous opinion has been adopted by many naturalists upon the Continent, even those conversant in comparative anatomy.

Whether the argonauta is really an internal shell, which I have asserted it to be, may possibly never be determined by direct proofs, as the animal belonging to it has not been met with. The present observations are confined to the question of the probability of its being formed by the species of sepia frequently found in it; and the materials of the present Paper, which are furnished from the specimens of natural history collected in the late expedition to the Congo, enable me to prove, in contradiction to such an opinion, that the ova of this particular species of sepia are not those of an animal of the order vermes testacea, that live in water.

The young of all oviparous animals, while contained in the ovum, must have their blood aerated through its coats, but in the vermes testacea, if the shell were formed in the ovum, the process of aerating the blood, must be very materially interfered with, for this reason, the covering or shell of the egg first drops off, and the young is hatched before the shell of the animal is formed; this I have seen take place in the eggs

of the garden snail, but in the testacea that live in water, the young requires some defence in the period, between the egg being hatched, and the young acquiring its shell, which is not necessary in those that live on land; for this purpose, the ova are enclosed in chambers of a particular kind.

This camerated nidus in the larger animals of this tribe, must be familiar to all naturalists, since specimens in a dried state, containing the young shells completely formed, are to be met with in collections of natural history; but I am not aware that all the purposes for which such a nidus is supplied by nature, have ever been explained.

I have been informed by a friend, who while in the East Indies saw the chank (a shell belonging to the same genus with the *voluta pyrum* of Linnæus,) shed its eggs, that the animal discharged a mass of mucus, adapted to the form of the lip of the shell, and several inches in length; this rope of eggs, enclosed in mucus at the end which is last disengaged, was of so adhesive a nature, that it became attached to the rock, or stone, on which the animal deposited it. As soon as the mucus came in contact with the salt water, it coagulated into a firm membranous structure, so that the eggs became enclosed in membranous chambers, and the nidus having one end fixed and the other loose, was moved by the waves, and the young in the eggs, had their blood aerated; when the young were hatched, they remained defended from the violence of the waves, till their shells had acquired strength.

What passes under the sea, few naturalists can be so fortunate as to have an opportunity of observing, and although what I have stated was communicated to me by an eye witness, it required confirmation, as well as an opportunity

of examining the nidus, before I could give it my assent. Since that time, I have procured from my friend Mr. LEE, the Botanist, of Hammersmith, a portion of a camerated nidus brought from South Carolina, containing shells of an univalve, not very different from the chanks of the East Indies. This nidus is represented in the annexed drawing. (Pl. XIII. fig. 7.)

I have also, which is still more satisfactory, seen the camerated nidus of the helix janthina. This animal not living at the bottom of the sea, like the vermes testacea in general, deposits its ova upon its own shell, if nothing else comes in its way; one of the specimens of the shell of the janthina, caught in the voyage to the Congo, fortunately has the ova so deposited, as will be seen in the annexed drawings made by Mr. BAUER, who was so pleased with the appearance the parts put on in the field of the microscope, that he was desirous of making a representation of them. (Pl. XIII. fig. 1, 2, 3, 4, 5, 6.)

In this instance, the ova are single, but in other tribes, several ova are contained in one chamber. In the land snail, the eggs have no such nidus. The following observations respecting them, were made in the year 1773, the first year that I was initiated in comparative anatomy, under Mr. HUNTER. He kept snails to ascertain their mode of breeding, and the notes that were made at the time in my own hand writing, I now copy.

August 5, 1773. A snail laid its eggs, and covered them over with earth; Mr. HUNTER took one out and examined it; the egg was round, its covering strong, and of a white colour, with a degree of transparency; it had no yelk; a small speck was observable with a magnifying glass in the transparent contents.

On the 9th no apparent change had taken place. On the 11th the speck had enlarged, but was too transparent to admit of its form being distinguished; upon moving the speck it fell out of its place.

On the 12th the embryo was indistinctly seen.

On the 15th the embryo filled ¼ part of the egg, but the different parts were still indistinct.

On the 18th the body of the embryo had become larger, and the covering thicker.

On the 19th, the coverings or shells of all the eggs were more or less dissolved, so much so that Mr. HUNTER thought all the eggs were rotting, and the whole brood of young would be lost.

On the 20th, the young were hatched, and the shells completely formed.

On the 23d, when the young snails were put in water, their bodies came out of the shell, as in full grown snails.

On the 24th, they all deserted their nests.

The specimens of the sepia found in the argonaut shell, which was caught by Mr. CRANCH, in this expedition to the Congo, had deposited some of its eggs in the involuted part of the shell, and the animal being fortunately caught in the shell, identified the eggs to belong to it; (Pl. XIV.) they are united together by pedicles, like the eggs of the sepia octopus, and in all other respects resemble them; they differ from those of the helix janthina and the other vermes testacea, that live in water, in having no camerated nidus, and in having a very large yelk to supply the young with nourishment, after they are hatched.

Upon these grounds, this animal must be resolved into a species of sepia, an animal which has no external shell,

Fig. 1.

Fig. 2.

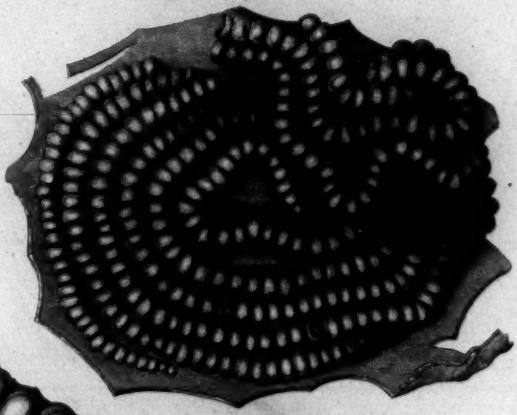


Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

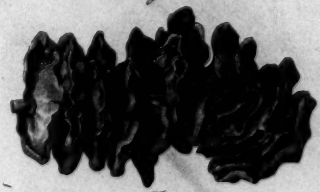


Fig. 7.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 5.

Fig. 4.

Fig. 6.

Fig. 7.

Franc. Bauer del.

J. Basire sc.

and only uses the shell of the argonaut, when it occasionally gets possession of one.

Some naturalists, unacquainted with comparative anatomy, have asserted that in these eggs they saw the argonaut shell partly formed; they must have mistaken the yelk, which will be seen in the drawing to be unusually large, for the new shell.

Explanation of the Plates.

Plate XIII.

Fig. 1. The shell of the helix janthina, with the ova in its camerated nidus, attached to it; magnified twice in diameter.

Fig. 2. A portion of the nidus magnified 12 times in diameter.

Fig. 3. A string of the same nidus magnified 25 times in diameter.

Fig. 4. Two of the same ova and one empty chamber, magnified 50 times in diameter.

Fig. 5. One of the same ova, and

Fig. 6. The same slightly bruised, both magnified 50 times in diameter.

Fig. 7. A portion of the camerated nidus, in a dried state, belonging to the ova of a univalve from South Carolina, of the natural size.

Plate XIV.

Fig. 1. The shell of the argonauta, with the ova of the octopus deposited in it, magnified twice in diameter.

Fig. 2. A cluster of the same ova, as they are seen when immersed in water, magnified 12 times in diameter.

Fig. 3. One of the same ova with its pellicle, magnified 25 times in diameter.

Fig. 4. The yelk of the egg.

Fig. 5. A transversal section of the same.

Fig. 6. A longitudinal section of the same. The three preceding figures are magnified 50 times in diameter.

Fig. 7. A collapsed egg, as seen when taken out of the water, magnified 25 times in diameter.

XXIV. *Astronomical observations and experiments tending to investigate the local arrangement of the celestial bodies in space, and to determine the extent and condition of the Milky Way.*

By Sir William Herschel, *Knt. Guelp. LL. D. F. R. S.*

Read June 19, 1817.

THE construction of the heavens, in which the real place of every celestial object in space is to be determined, can only be delineated with precision, when we have the situation of each heavenly body assigned in three dimensions, which in the case of the visible universe may be called length, breadth, and depth; or longitude, latitude, and Profundity.

The angular positions of the stars and other celestial objects, as they are given in astronomical catalogues, and represented upon globes, or laid down in maps, enable us, in a clear night, to find them by the eye or to view them in a telescope; for, in order to direct an instrument to them, a superficial place consisting of only two dimensions is sufficient; but although the line in which they are to be seen is thus pointed out to us, their distance from the eye in that line remains unknown; and unless a proper method for obtaining the profundity of objects can be found, their longitude and latitude will not enable us to assign their local arrangement in space.

With regard to objects comparatively very near to us, astronomers have completely succeeded by the method of parallaxes. The distance of the sun; the dimensions of the

orbits of the planets and of their satellites; the diameters of the sun, the moon, and the rest of the bodies belonging to the solar system, as well as the distances of comets, have all been successfully ascertained. The parallax of the fixed stars has also been an object of attention; and although we have hitherto had no satisfactory result from the investigation, the attempt has at least so far succeeded as to give us a most magnificent idea of the vast expansion of the sidereal heavens, by showing that probably the whole diameter of the earth's orbit, at the distance of a star of the first magnitude, does not subtend an angle of more than a single second of a degree, if indeed it should amount to so much; with regard to more remote objects, however, such as the stars of smaller size, highly compressed clusters of stars and nebulæ, the parallactic method can give us no assistance.

I. *Of the local situation of the stars of the heavens.*

The superficial situation of the stars having already been carefully assigned in the catalogues of astronomers, it will be proper to examine how far the arrangement of the stars into a certain order of magnitudes can assist us to determine their local situation.

When we look at the heavens in a clear night, and observe the different lustre of the stars, we are impressed with a certain idea of their different magnitudes; and when our estimation is confined to their appearance only, we shall be justified in saying, for instance, that Arcturus is larger than Aldebaran; the principle on which the stars are classed is, therefore, entirely founded on their apparent magnitude, or

brightness. Now, as it was thought convenient to arrange all the stars which in fine weather may be seen by the eye into seven classes, the brightest were called of the first, and the rest according to their gradually diminishing lustre, of the 2d, 3d, 4th, 5th, 6th, and 7th magnitudes. Then, since it is evident that we cannot mean to affirm that the stars of the 5th, 6th, and 7th magnitudes are really smaller than those of the 1st, 2d, or 3d, we must ascribe the cause of the difference in the apparent magnitudes of the stars to a difference in their relative distances from us; and on account of the great number of stars contained in each class, we must also allow that the stars of each succeeding magnitude, beginning from the first, are one with another farther from us than those of the magnitude immediately preceding. It may therefore be said, that since in our catalogues the magnitudes are added to the two dimensions which give the superficial place of the stars, we have also at least a presumptive value of the third dimension; but admitting that the naked eye can see stars as far from us as those of the seventh magnitude, this presumptive value, which can only point out their relative situation, will afford us no information about the real distance at which they are placed.

II. *Of a standard by which the relative arrangement of the stars may be examined.*

It is evident, that when we propose to examine how the stars of the heavens are arranged, we ought to have a certain standard of reference; and this I believe may be had by comparing their distribution to a certain properly modified equality of scattering. Now, the equality I shall here propose,

does not require that the stars should be at equal distances from each other; nor is it necessary that all those of the same nominal magnitude should be equally distant from us. It consists in allotting a certain equal portion of space to every star, in consequence of which we may calculate how many stars any given extent of space should contain. This definition of equal scattering agrees so far with observation, that it admits, for instance, Sirius, Arcturus, and Aldebaran to be put into the same class, notwithstanding their very different lustre will not allow us to suppose them to be at equal distances from us; but its chief advantage will be, that instead of the order of magnitudes into which our catalogues have arranged the stars, it will give us an order of distances, which may be used for ascertaining the local distribution of the heavenly bodies in space.

To explain this arrangement, let a circle be drawn with any given radius about the point S fig. 1, Plate XV. and with 3, 5, 7, 9, &c. times the same radius draw circles, or circular arcs, about the same centre. Then if a portion of space equal to the solid contents of a sphere, represented by the circle S, be allotted to each star, the circles, or circular arcs drawn about it will denote spheres containing the stars of their own order, and of all the orders belonging to the included spheres, and on the supposition of an equality of scattering, the number of stars of any given order may be had by inspection of the figure, which contains all the numbers that are required for the purpose; for those in front of the diagram express the diameters of spherical figures. The first row of numbers enclosed between the successive arcs, are the cubes of the diameters; the next column expresses the order

of the central distances; and the last gives the difference between the cube numbers of any order and the cube of the next enclosed order.

The use to be made of these columns of numbers is by inspection to determine how many stars of any particular order there ought to be if the stars were equally scattered. For instance, let it be required how many stars there should be of the 4th order. Then No. 4, in the column of the orders points out a sphere of nine times the diameter of the central one, and shows that it would contain 729 stars; but as this sphere includes all the stars of the 3d, 2d, and 1st order as well as the sun, their number will be the sum of all the stars contained in the next inferior sphere amounting to 343; which being taken from 729 leaves 386 for the space allotted to those of the 4th order of distances.

III. *Comparison of the order of magnitudes with the order of distances.*

With a view to throw some light upon the question, in what manner the stars are scattered in space, we may now compare their magnitudes, as we find them assigned in Mr. BODE's extensive catalogue of stars, with the order of their distances which has been explained.

The catalogue I have mentioned contains 17 stars of the 1st magnitude; but in my figure of the order of the distances we find their number to be 26.

The same catalogue has 57 stars of the 2d magnitude; but the order of distances admits 98.

Of the third magnitude the catalogue has 206, and the order of distances will admit 218.

The number of the stars of the 4th magnitude is by the catalogue 454, and by the order of distances 386.

Before I proceed, it may be proper to remark, that, by these four classifications of the stars into magnitudes, it appears already, that, on account of the great difference in the lustre of the brightest stars, many of them have been put back into the second class; and that the same visible excess of light has also occasioned many of the stars of the next degree of brightness to be put into the third class; but the principle of the visibility of the difference in brightness would have less influence with the gradually diminishing lustre of the stars, so that the number of those of the third magnitude would come nearly up to those of the third distance. And as the difference in the light of small stars is less visible than in the large ones, we find that the catalogue has admitted a greater number of stars of the 4th magnitude than the 4th order of distances points out; this may, however, be owing to taking in the stars that were thrown back from the preceding orders; and a remarkable coincidence of numbers seems to confirm this account of the arrangement of the stars into magnitudes. For the total number of the catalogued stars of the 1st, 2d, 3d, and 4th magnitudes, with the addition of the sun, is 785; and the number contained in the whole sphere of the 4th distance is 729.

Now the distinguishable difference of brightness becoming gradually less as the stars are smaller, the effect of the principle of classification will be, as indeed we find it in the 5th, 6th, and 7th classes, that fainter stars must be admitted into them than the order of distances points out.

The catalogue contains 1161 stars of the 5th magnitude, whereas the 5th order of distances has only room for 602.

Of the 6th magnitude the catalogue contains not less than 6103 stars, but the 6th order of distances will admit only 866.

And lastly, the same catalogue points out 6146 stars of the 7th magnitude, while the number of stars that can be taken into the 7th order of distances is only 1178.

The result of this comparison therefore is, that if the order of magnitudes could indicate the distance of the stars, it would denote at first a gradual, and afterwards a very abrupt condensation of them; but that, considering the principle on which the stars are classed, their arrangement into magnitudes can only apply to certain relative distances, and show that taking the stars of each class one with another, those of the succeeding magnitudes are farther from us than the stars of the preceding order.

IV. *Of a criterion for ascertaining the Profundity, or local situation of celestial objects in space.*

It has been shown that the presumptive distances of the stars pointed out by their magnitudes can give us no information of their real situation in space. The statement, however, that one with another the faintest stars are at the greatest distance from us, seems to me so forcible, that I believe it may serve for the foundation of an experimental investigation.

It will be admitted, that the light of a star is inversely as the square of its distance; if therefore we can find a method

by which the degree of light of any given star may be ascertained, its distance will become a subject of calculation. But in order to draw valid consequences from experiments made upon the brightness of different stars, we shall be obliged to admit, that one with another the stars are of a certain physical generic size and brightness, still allowing that all such deviations may exist, as generally take place among the individuals belonging to the same species.

There may be some difference in the intrinsic brightness of starlight: that of highly coloured stars may differ from the light of the bluish white ones; but in remarkable cases allowances may be made.

With regard to size, or diameter, we are perhaps more liable to error; but the extensive catalogue which has already been consulted, contains not less than 14,144 stars of the seven magnitudes that have been adverted to; it may therefore be presumed that any star promiscuously chosen for an experiment, out of such a number, is not likely to differ much from a certain mean size of them all.

At all events it will be certain that those stars the light of which we can experimentally prove to be $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, $\frac{1}{36}$, and $\frac{1}{49}$ of the light of any certain star of the 1st magnitude, must be 2, 3, 4, 5, 6, and 7 times as far from us as the standard star, provided the condition of the stars should come up to the supposed mean state of diameter and lustre of the standard star, and of this, when many equalisations are made, there is at least a great probability in favour.

V. *Of the equalization of starlight.*

In my sweeps of the heavens, the idea of ascertaining the

Profundity of space to which our telescopes might reach, gave rise to an investigation of their space penetrating power; and finding that this might be calculated with reference to the extent of the same power of which the unassisted eye is capable, there always remained a desideratum of some sure method by which this might be ascertained.

Of various experiments I have long ago tried, the equalisation of starlight, which about four years ago I began to put into execution, appeared to be the most practicable. A description of the apparatus and the method of making use of it is as follows.

Of ten highly finished mirrors I selected two of an equal diameter and focal length, and placed them in two similarly fitted up seven feet telescopes. When they were completely adjusted, I directed them both, with a magnifying power of 118, to the same star, for instance, Arcturus: and upon trial I found the light not only of this, but of every other star to which they were directed, perfectly equal in both telescopes.

The two instruments, when I viewed the stars, were placed one a little before the other, and so near together that it would require little more than one second of time to look from one into the other. This convenient situation of the instruments is of great importance. The impression of the light made by the view of one star should be succeeded as soon as possible by the view of the other; and these alternate inspections should also be many times repeated, in order to take away some little advantage which the last view of a bright object has over that immediately preceding.

In comparing the light of one star with that of another, I laid it down as a principle, that no estimation but that of per-

fect equality should be admitted; and as the equal action of the instruments was now ascertained, I calculated the diameters of several apertures to be given to one of the telescopes as a standard, so that the other, called the equalising telescope, might be employed, with all its aperture unconfined, to examine a variety of stars, till one of them was found whose light was equal to that of the star to which the standard telescope was directed.*

In order to be sufficiently accurate in the calculation of the diameter of the limiting apertures, I thought it necessary to take into consideration not only the obstruction of incident light occasioned by the interposition of the small mirror, but also of the arm to which it is fastened, and proceeded as follows:

If A be the diameter of the large mirror; b that of the small one; $\frac{A-b}{2}$ the length of the arm; t its thickness; π the circumference, diameter being unity; x an assumed quantity for finding the correction; A′ the aperture corrected for the interposition of the arm; L the light of the equalising telescope; p the proportion of the light required for the standard telescope; D the diameter of an aperture to give that light; D′ the diameter corrected for the interposition of the arm.

Then will the diameters of the limiting apertures be had by the following equations. $\frac{A-b}{2} \times t = \pi A x$; $\frac{A-b}{\pi A} \times t = 2x$; $A - 2x = A'$; $A'' - b^2 = L$; $pL = D^2 - b^2$; $\sqrt{pL + b^2} = D$; $\frac{D-b}{\pi D} \times t = 2y$; $D + 2y = D'$ the required diameter.

* I preferred the limitation of the light by circular apertures to the method of obtaining it by the approach or recess of two opposite rectangular plates, in order to avoid the inflections which take place in the angles.

In the calculation of a set of apertures for the intended purpose, I admitted none that gave less than $\frac{1}{4}$ of the light; for by a greater contraction of the aperture of the mirror, an increase of the spurious diameters would render a judgment of equality liable to deception;† when therefore a star of the third order of distances was to be found, I rejected the direct way of reducing a star of the first order to $\frac{1}{9}$ of its light, but selected a star previously ascertained to be of the second order; and by taking $\frac{1}{4}$ of its light, the equalising telescope, with all its light, was used to examine all such stars as appeared likely to give the required equality, till one of them was found; nor was it necessary to have a great number of limiting apertures, as it soon appeared that with eight or ten of them I could have many different gradations of light, which would ascertain even fractional degrees, and reach as far as the stars of any order of distances I could expect to be visible to the unassisted eye.

This method of equalising the light of the stars, easy as it may appear, is nevertheless subject to great difficulties; for as the brightness of a star is affected by its situation, with regard to the ambient light of the heavens, the stars to be equalised should, if possible, be in nearly the same region. When the sun is deep under the horizon, this is however not of so much consequence as the altitude of the star to be equa-

† This was fully proved by the following experiment. July 27, 1813. I viewed Arcturus in a 10 feet reflector; first with all its light; next with circular diaphragms, which confined its aperture to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{25}$, and $\frac{1}{35}$ of it; but I found that the different spurious diameters, arising from the smallness of the apertures, made estimations of what is generally called the magnitude of the stars impossible.

See also experiments on the spurious diameters of the celestial objects, Phil. Trans. for 1805, page 40.

lised, which ought to be as nearly as possible equal to that of the standard star. At great elevations some difference in the altitudes of the stars to be equalised may be admitted; but, if they are far from each other, the circumstance of the equal illumination of the heavens, and the equal clearness of the air, must still be attended to.

VI. *Of the extent of natural vision.*

The method of equalising star light may be rigorously applied to ascertain the extent of natural vision; for in this case it will not be required that the star on which the experiment is tried, should be of the same size or diameter with the standard star; nor is it necessary that the intrinsic brightness of the light of the two stars should be the same in both. It will be sufficient, that the star we choose for an equalisation is one of the smallest that are still visible to the natural eye. It is also to be understood that, till we can have a well ascertained value of the parallax of any one star of the first magnitude, the extent of natural vision can only be given in a measure of which the distance of the standard star is the unit.

The following equalisations were made in August and December 1803, and February 1814, and are given as a specimen of the method I have pursued.

Taking Arcturus for the standard of an experiment, I directed the telescope, with one quarter of its light, upon it; while the equalising telescope, with all its light, was successively set upon such stars as I supposed might be at double the distance of the standard star; which, as Arcturus is a star

of the first magnitude, I expected to find among those of the second.

The first I tried was β (FL. 58) Pegasi, but I found it not quite bright enough.

The light of α Andromedæ, which next I tried, was nearly equalised to that of Arcturus; and the observation being repeated on a different night gave it equal.

Now as in these experiments the standard star is supposed to be one of the first order of distances, it follows that, if Arcturus were put at twice its distance from us, it would then appear like α Andromedæ, as a star of the 2d magnitude, and would also at the same time be really a star of the 2d order of distances.

In order to obtain some other stars whose light might be equalised by one quarter the light of Arcturus, I tried many different ones, and found among them α Polaris, γ Ursæ, and δ Cassiopeæ. These stars therefore may also be put into the class of those whose light is equal to the stars of the second order of the distance of Arcturus.

For the purpose of ascertaining the extent of natural vision, it will not be necessary here to give the equalisation of stars of the 3d, 5th, 6th, or 7th order of distances; but taking now the light of one of the stars of the 2d order of distances for a standard, I tried many that might be expected to have the required light, and found that when α Andromedæ, with its light reduced to one quarter, was in the standard telescope, the equalising one gave μ (FL. 48.) Pegasi for a star of the 4th order of distances. That is to say, the equalisation proved that, if Arcturus were placed at four times its distance from

us, we should see it as a star of the 4th magnitude, and also as one of the 4th order of distances.

Proceeding in the same manner with μ Pegasi taken as a standard, I found that its light reduced to $\frac{1}{4}$ was equal to that of q (Fl. 70.) Pegasi, when seen in the equalising telescope; and that consequently Arcturus, removed to 8 times its present distance from us, would put on the appearance of a star which in our catalogues is called of the 5th or 6th magnitude, but which would in fact be of the 8th order of distances.

As the foregoing experiments can only show that a star of the light of Arcturus might be removed to 8 times its distance, and still remain visible to the naked eye as a star of between the 5th and 6th magnitude; it will be proper to take also other stars of the first magnitude for the original standards.

For instance, if we begin from Capella as the standard star, we may with $\frac{1}{4}$ of its light equalise β Aurigæ and β Tauri, which stars will therefore be of the 2d order of distances. With $\frac{1}{4}$ of the light of β Tauri we equalise ζ Tauri and ι Aurigæ; they will then be of the 4th order. With $\frac{1}{4}$ of the light of ι Aurigæ we can equalise e Persei and H Geminorum which will be of the 8th order. And with $\frac{1}{2\frac{1}{2}}$ of the light of H Geminorum we equalise d Geminorum, which makes it a star of the 10th order. That is to say, if Capella were successively removed to 2, 4, 8 and 10 times the distance at which it is from us, it would then have the appearance of the stars which have been named.

A similar deduction may be made from α Lyræ, as $\frac{1}{4}$ of its light equalises it with β Tauri; for it will be α Lyræ 1, β Tauri 2, ι Aurigæ 4, H Geminorum 8, and d Geminorum 10: the

numbers annexed to the stars expressing their orders of distances in terms of the distance of α Lyræ from us.

To find stars of the intermediate orders of distances, the following Table gives the proportional light that should be used with the star which is made the standard; for instance, a star of the 2d order of distances, with $\frac{4}{9}$ of its light, will equalise a star of the 3d order; $\frac{9}{25}$ of the light of a star of the 3d order of distances will give one of the 5th order, and so on.

A star of the order of distances.	With the proportion of its light.	Gives one of the order of distances.
1	$\frac{1}{4}$	2
2	$\frac{4}{9}$	3
	$\frac{1}{4}$	$4\frac{1}{2}$
3	$\frac{9}{25}$	5
	$\frac{1}{4}$	6
4	$\frac{16}{49}$	7
	$\frac{1}{4}$	8
5	$\frac{25}{81}$	9
	$\frac{1}{4}$	10
6	$\frac{36}{121}$	11
	$\frac{1}{4}$	12

Some other proportions of light useful for fractional distances are $\frac{9}{16}$, $\frac{16}{25}$, $\frac{16}{49}$, $\frac{64}{81}$, and $\frac{100}{121}$.

The results of equalisations that are made with different standard stars, may be connected together by an equalisation of the standards; by which means many different sets may be brought to support each other. For instance, Capella with $\frac{16}{49}$ of its light is of an equal lustre with Procyon, which therefore is of the $1\frac{1}{2}$ order of Capella, and Sirius with

$\frac{1}{45}$ of its light is also of an equal lustre with Procyon, which consequently, with regard to Sirius, is of the $1\frac{1}{4}$ order; then, by compounding, it follows that Capella to Sirius is a star of the $1\frac{1}{2}$ order, and from this we obtain the following series. Sirius 1, Capella $1\frac{1}{2}$, Procyon $1\frac{3}{4}$, β Tauri 3, ι Aurigæ 6, H Geminorum 12, and *d* Geminorum 15. By this connection we shall be able to obtain an equalisation of the same ultimate star with all the standards; for if Sirius must be removed to the 15th order, to appear as faint as *d* Geminorum; and if Capella, and also α Lyræ must be removed to the 10th order of distances to appear as faint as the same star, then any star of the size and brightness of Sirius, Capella, and α Lyræ must generally appear as faint as *d* Geminorum, when it is removed to nearly 12 times its distance; and the more stars of the first order are admitted in these general equalisations reduced to the same faint star, the more will the probability of the result be extended. Now as *d* Geminorum is a star of the 6th magnitude, we may expect that a still fainter visible star will give a somewhat greater extent to the reach of the natural eye, if however I take its vision, including other stars of the 1st magnitude, to extend to the 12th order of distances, there will probably be no material error, at least none but what a diligent astronomer, who is provided with the necessary apparatus, may correct by observation.

But the extent of natural vision is not limited to the light of solitary stars only; the united lustre of a number of them will become visible when the stars themselves cannot be seen. For instance, the milky way; the bright spot in the sword handle of Perseus; the cluster north of η and H Geminorum; the cluster south of Fl. 6 and 9 Aquilæ; the clus-

ter south of η Herculis, and the cluster north preceding ε Pegasi. But their distances cannot be ascertained by the method of equalising starlight: their probable situation in space may however be deduced from telescopic observations.

To these very faintly visible objects may be added two of a different nature, namely, the nebulosity in the sword of Orion, and that in the girdle of Andromeda.

VII. *Of the extent of telescopic vision.*

The powers of telescopes to penetrate into the Profundity of space is the result of the quantity of light they collect and send to the eye in a state fit for vision. The method of calculating the quantity of this power has been fully explained in a Paper read before the Royal Society, November 21, 1799; and the formulæ which have been given in that Paper have already been applied to show to what extent this power has been carried in the telescopes I used for astronomical observations. The calculated results, however, give this power only in reference to that of natural vision, and the uncertainty in which we were left with regard to its extent, was equally thrown over that of telescopic vision.

The equalisation of starlight, when carried to a proper degree of accuracy, will do away the cause of the error to which the telescopic extent of vision has been unavoidably subject, we may therefore safely apply this vision to measure the Profundity of sidereal objects that are far beyond the reach of the natural eye; but for this purpose the powers of penetrating into space of the telescopes that are to be used must be reduced to what may be called gaging powers; and as

the formula $\frac{\sqrt{x \cdot A^2 - b^2}}{a}$ gives the whole quantity of the space penetrating power, a reduction to any inferior power p, may be made by the expression $\sqrt{\frac{p^2 a^2}{x} + b^2} = A$; when the aperture is then limited to the calculated value of A, the telescopes will have the required gaging power. Or we may prepare a regular set of apertures to serve for trials, and find the gaging powers they give to the telescope by the original formula.

In the formula by which the required apertures for the gaging powers were calculated, a has been put equal to two tenths of an inch, and to show that this assumption is founded upon observation, I give the following extract from my astronomical Journal.

Dec. 27, 1801. I looked at α Lyræ with one eye shut and the other guarded by a slip of brass with holes of various sizes in it. Through the hole which was 0.28 inch in diameter, I saw the star just as well as without the limiting diaphragm, which shows that the opening of the pupil of the eye does not exceed 0.28 inch.

I tried the same star through 0.24 and still saw it equally well. I tried next 0.21 and still saw it as well.

The slip of brass was held as close to the eye as possible. The next I tried was 0.17 in diameter, and through this I could perceive a small deficiency of light, so that the opening of the pupil exceeds 0.17 inch. The night is hardly dark enough yet for great accuracy.

Having been out long in the dark, and trying the same

* See Phil. Trans. for 1800, page 66.

experiment upon many different large and small stars, they all concur to show that 0.21 does not sensibly stop any light; but that less does certainly render the object rather less luminous; so that the opening may be put at two-tenths of an inch in my eye.

VIII. *Application of the extent of natural and telescopic vision to the probable arrangement of the celestial bodies in space.*

When the extent of natural and telescopic vision is to be applied to investigate the distance of celestial objects, the result can only have a high degree of probability; for it will then be necessary to admit a certain physical generic size and brightness of the stars. But when two hypotheses are proposed to explain a certain phenomenon, that which will most naturally account for it ought to be preferred as being the most probable. Now as the different magnitudes of the stars may be ascribed to a physical difference in their size and lustre, and may also be owing to the greater distance of the fainter ones, we cannot think it probable that all those of the 5th, 6th and 7th magnitude, should be gradually of a smaller physical construction than those of the 1st, 2d, and 3d; but shall, on the contrary, be fairly justified in concluding that, in conformity with all the phenomena of vision, the greater faintness of those stars is owing to their greater distance from us. The average size and brightness of several stars of the first magnitude being also taken as a standard, in the manner that has been shown, the conclusion drawn from different series of equalisations will support one another; so that we shall be able to say a distant celestial object is so far from us,

provided the stars of which it is composed are of a size and lustre equal to the size and lustre of such stars as Sirius, Arcturus, Capella, Lyra, Rigel, and Procyon, &c.

I proceed now to consider some conclusions that may be drawn from a known extent of natural vision, a very obvious one of which is, that all the visible stars are probably contained within a sphere of the 12th order of distances. Now as on the principle of equal scattering, we should see about 1569 5 of them, it may be remarked that the stars of the catalogue, including all those of the 7th magnitude, amount to 14144, which agrees sufficiently well with the calculated number; but the next inference is, that if they were equally scattered, there would be 2402 of the 10th, 2906 of the 11th, and 3458 of the 12th order of distances, which added together amount only to 8766, whereas the number of stars of the 6th and 7th magnitudes that must come into these three orders, is not less than 12249, which would indicate that the stars in the higher order of distances are more compressed than they are in the neighbourhood of the sun; but from astronomical observations, we also know that the stars of the 6th and 7th magnitude are very sparingly scattered over many of the constellations, and that consequently the stars which belong to the 10th, 11th, and 12th order of distances, are not only more compressed than those in the neighbourhood of the sun, but that moreover their compression in different parts of the heavens must be very unequal.

IX. *Of the construction and extent of the milky way.*

Of all the celestial objects consisting of stars not visible to the eye, the milky way is the most striking; its general

appearance, without applying a telescope to it, is that of a zone surrounding our situation in the solar system, in the shape of a succession of differently condensed patches of brightness, intermixed with others of a fainter tinge.

To enumerate a partial series of them, we have a very bright patch under the arrow of Sagittarius; another in the Scutum Sobiescii; between these two there are three unequally bright places; north preceding α β and γ Aquilæ is a bright patch; between Aquila and the Scutum are two very faint places; a long faint place follows the shoulder of Ophiucus; near β Cygni is a bright place; near γ is another, and a third near α. A smaller brightish place follows in the succession of the milky way, and a large one towards Cassiopea. A faint place is on one side; a second towards Cassiopea, and a third is within that constellation; a very bright place is in the sword handle of Perseus; and α and γ Cassiopeæ inclose a dark spot.

The breadth of the milky way appears to be very unequal. In a few places it does not exceed five degrees; but in several constellations it is extended from ten to sixteen. In its course it runs nearly 120 degrees in a divided clustering stream, of which the two branches between Serpentarius and Antinous are expanded over more than 22 degrees.

That the sun is within its plane may be seen by an observer in the latitude of about 60 degrees; for when at 100 degrees of right ascension the milky way is in the east, it will at the same time be in the west at 280; while in its meridional situation it will pass through Cassiopea in the Zenith, and through the constellation of the cross in the Nadir.

From this survey of the milky way by the eye I shall now

proceed to show what appears to be its construction by applying to it the extent of telescopic vision; but as I had prepared a gradually increasing series of reductions of the space-penetrating powers of my instruments for the purpose of measuring the Profundity of sidereal objects not visible to the eye, which I have called gaging powers, it will be necessary to give the following account of it.

From the formula which has been given, I calculated a set of apertures, which by limiting the light of the finder of my seven feet reflector would reduce its space-penetrating power to the low gaging powers 2, 3, and 4. I then limited in the same manner the space-penetrating power of my night glass, by using calculated apertures such as would give the gaging powers 5, 6, 7 and 8. From the space-penetrating power of the 7 feet reflector, I obtained by limitation the successive gaging powers 9, 10 and upwards to 17. And lastly, by limiting the space-penetrating power of my 10 feet reflector, I carried the gaging powers from 17 to 28.

For the purpose of trying these powers, I selected the bright spot in the sword handle of Perseus, as being probably a protuberant part of the milky way, in which it is situated. Its altitude at the time of observation was about 30 degrees, and no star in it was visible.

In the finder with the gaging power 2, I saw many stars; and admitting the eye to reach to stars at the distance of the 12th order, we may conclude that the small stars which were visible with this low power, are such as contribute to the brightness of the spot, and that their situation is probably from between the 12th to the 24th order of distances; at least we are certain, that if stars of the size and lustre of

Sirius, Arcturus, Capella, &c. were removed into the Profundity of space which I have mentioned, they would then appear like the stars I saw with the gaging power of the finder. I then changed this power from 2 to 3, and saw more stars than before; and changing it again from 3 to 4, a still greater number of them became visible. The situation of these additional stars was consequently between the 24th, 36th, and 48th order of distances.

With the gaging power 5 of the night glass I saw a great number of stars; with 6, more stars and whitishness became visible; with 7, more stars with resolvable whitishness were seen; and with 8, still more. The stars that gradually made their appearance, therefore, were probably scattered over the space between the 48th and 96th order of distances.

In the 7 feet reflector, with the gaging powers 9 and 10, I saw a great number of stars; with 11 and 12, a greater number of stars and resolvable whitishness were seen; with 13 and 14, the number of visible stars was increased, and was so again with 15; with 16 and 17 in addition to the visible stars, there were many too faint to be distinctly perceived. These gages therefore extend the space over which the additional stars were scattered from the 96th to the 204th order of distances.

With a 10 feet reflector, reduced to a gaging power of 18, I saw a great number of stars: they were of very different magnitudes, and many whitish appearances were so faint that their consisting of stars remained doubtful. The power 19, which next I used, verified the reality of several suspected stars, and increased the lustre of the former ones. With 20, 22, and 25, the same progressive verifications of suspected

stars took place, and those which had been verified by the preceding powers, received subsequent additional illumination. With the whole space-penetrating power of the instrument, which is 28.67, the extremely faint stars in the field of view acquired more light, and many still fainter suspected whitish points could not be verified for want of a still higher gaging power. The stars which filled the field of view were of every various order of telescopic magnitudes, and, as appears by these observations, were probably scattered over a space extending from the 904th to the 344th order of distances.

As the power of the 10 feet reflector could not reach farther into space, I shall have recourse to some of my numerous observations made with the 20 feet telescope. In addition to 863 gages already published,* above 400 more have been taken in various parts of the heavens, but with regard to these gages, which on a supposition of an equality of scattering were looked upon as gages of distances, I have now to remark that, although a greater number of stars in the field of view is generally an indication of their greater distance from us, these gages, in fact, relate more immediately to the scattering of the stars, of which they give us a valuable information, such as will prove the different richness of the various regions of the heavens.

July 30, 1785. Right ascension $19^h\ 4'$. Polar distance $87°\ 5'$. The milky way is extremely rich in stars that are too small for the gage.

Dec. 7, 1785. Right ascension $5^h\ 33'$. Polar distance $66°\ 6'$.

* See Phil. Trans. for 1785, p. 221.

There are about 66 stars in the field of view, and many more so extremely small as not to admit of being gaged.

Sept. 20, 1786. Right ascension $20^h\ 40'$. Polar distance $54°\ 36'$. There are about 80 stars in a quadrant, or 320 in the field of view, besides many more too small to be distinctly seen.

Oct. 14, 1787. Right ascension $21^h\ 57'$. Polar distance from $38°\ 18'$ to $38°\ 50'$. In this part of the heavens the large stars seem to be of the 9th and 10th magnitude. The small ones are gradually less till they escape the eye, so that appearances here favour the idea of a succeeding, more distant clustering part of the milky way.

Sept. 18, 1784. Right ascension $20^h\ 8'$. Polar distance from $70°\ 9'$ to $72°\ 49'$. The end of the stratum of the stars of the milky way cannot be seen.

By these observations it appears that the utmost stretch of the space-penetrating power of the 20 feet telescope could not fathom the Profundity of the milky way, and that the stars which were beyond its reach must have been farther from us than the 900dth order of distances.

I am far from limiting the milky way to the extent deduced from these observations; but as even the distance which has been stated may appear doubtful, I must repeat the argument which has been used with stars visible to the eye, but which now is greatly supported by telescopic vision. If the stars of the 5th, 6th, and 7th magnitudes cannot be supposed to be gradually of a smaller physical size and brightness than those of the 1st, 2d, and 3d, how much less can a supposition be admitted that would require that the stars, which by a long

series of gaging powers have been proved to make their gradual telescopic appearance from the 12th to the 900dth order of distances, should also be gradually of a different construction, with regard to physical size and brightness, from those which we see with the naked eye.

From the great diameter of the mirror of the 40 feet telescope we have reason to believe, that a review of the milky way with this instrument would carry the extent of this brilliant arrangement of stars as far into space as its penetrating power can reach, which would be to the 2300dth order of distances, and that it would then probably leave us again in the same uncertainty as the 20 feet telescope. When I made some sweeps of the heavens with the 40 feet telescope with a magnifying power of 370, I found it necessary to reduce the intended breadth of the sweep from one degree to 30 minutes, and the great length of time this would have taken up to examine only the ecliptic, to which I had directed the telescope, soon proved that by continuing to use this instrument for sweeping, I should have been obliged to neglect the necessary observations of the 20 feet telescope.

The following observations are extracted from my sweeps to support a few general remarks relating to the construction of the milky way.

Dec. 7, 1785. Right ascension 4^h 39′. Polar distance from 64° 0′ to 66° 12′. The straggling stars of the milky way seem now to come on gradually; most of them are small. Right ascension 4° 48′. They begin now to be intermixed with some large ones.

This observation proves that the telescopic breadth of the milky way, considerably exceeds the extent which in our

maps is assigned to it. In this situation it began to appear at 6 or 7 degrees from where it might have been expected to enter the telescope.

Aug. 21, 1791. Right ascension 18^h 59′. Polar distance from 84° 15′ to 86° 17′. The milky way comes on very suddenly, and is amazingly crowded with very small stars intermixed with many of several sizes.

By our maps this place is already within the limits of the milky way.

Jan. 1, 1786. Right ascension 5^h 17′. Polar distance from 89° 28′ to 91° 47′. Most of the stars are larger than usual, but the whole breadth of the sweep contains a great mixture of all sizes.

From the brightness of the stars we may conclude that the constellation of Orion to which the observation belongs, is one of those that are nearest to our own situation.

Dec. 27, 1786. Right ascension 6^h 49′. Polar distance from 87° 37′ to 89° 55′. From the appearance of the heavens in this neighbourhood, there is reason to suppose that there is a break or vacancy among the stars between our situation and the more remote parts of the milky way.

The place to which this observation refers is in the breast of Monoceros.

Oct. 14. 1787. Right ascension 22^h 14′. Polar distance from 35° 18′ to 38° 50′. It is very evident that in this part of the heavens there is some distance between us and the milky way, which is not equally scattered over with stars.

The situation of the place pointed out by the observation is near the crown of Cepheus.

Sept. 15, 1792. Right ascension 19^h 46′. Polar distance

52° 29′. There are 153 stars in a quadrant, or 612 in the field of view, and the whole breadth of the sweep, which is 2° 2′, is equally rich.

The gage was taken in the preceding branch of the milky way, in the neck of the Swan.

Aug. 22, 1792. Right ascension 19^h 35′. Polar distance 75° 5′. The field of view is extremely crowded, but the stars are too small and too numerous to be counted; there cannot be less than 100, or probably 150 in a quadrant. From some careful trials I suppose there were 150; this would give 600 stars in the field of view. The stars continue to be equally crowded throughout the whole breadth of the sweep, which was 2° 35′ till right ascension 19^h 54′, when there still were 440 in the field.

This gage was taken in the following branch of the milky way, in the wing of Aquila.

Sept. 18, 1784. Right ascension 20^h 48′. Polar distance from 54° 15′ to 57° 1′. This branch of the milky way is less rich than the preceding one.

The same sweep passed through both the branches, and the observation relates to a place in the following wing of the Swan.

Oct. 19, 1788. Right ascension 21^h 13′. Polar distance from 48° 35′ to 46° 13′. The milky way is very rich, but the stars are very unequally scattered.

This observation belongs to the tail of the Swan.

Nov. 26, 1788. Right ascension 23^h 40′. Polar distance 29° 13′. The milky way is very rich, but the stars are very unequally scattered. The stars are clustering. Right ascension 0^h 14′. Polar distance 29° 51′. Clustering small stars

considerably rich. Right ascension 0^h 27′. Polar distance 30° 5′. Clustering small stars.

These observations belong to a place preceding the back of Cassiopea's chair.

Dec. 27, 1786. Right ascension 6^h 36′. Polar distance 88° 5′. Clustering stars of the milky way, almost close enough, and so far separated from the rest of the stars as to be called a cluster, but still evidently joining to them.

This observation of the clustering of the stars of the milky way relates to a place that precedes the breast of Monoceros.

X. *Concluding Remarks.*

What has been said of the extent and condition of the milky way in several of my papers on the construction of the heavens, with the addition of the observations contained in this attempt to give a more correct idea of its profundity in space, will nearly contain all the general knowledge we can ever have of this magnificent collection of stars. To enter upon the subject of the contents of the heavens in the two comparatively vacant spaces on each side adjoining the milky way, the situation of globular clusters of planetary nebulæ, and of far extended nebulosities, would greatly exceed the compass of this Paper; I shall therefore only add one remarkable conclusion that may be drawn from the experiments which have been made with the gaging powers.

In fig. 2, Plate XV. let a circle, drawn with the radius of the 12th order of distances, represent a sphere containing every star that can be seen by the naked eye; then, if the breadth of the milky way were only 5 degrees, and if its profundity did not exceed the 900th order of distances, the

Fig. 1.

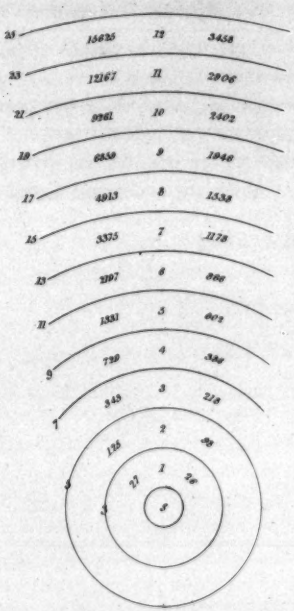

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Fig. 2.

two parallel lines in the figure, representing the breadth of the milky way, will, on each side of the centre of the inclosed circle, extend to more than the 39th order of distances.

From this it follows, that not only our sun, but all the stars we can see with the eye, are deeply immersed in the milky way, and form a component part of it.

WILLIAM HERSCHEL.

Slough, near Windsor,
May 10, 1817.

XXV. *Some account of the nests of the Java swallow, and of the glands that secrete the mucus of which they are composed.* By *Sir* EVERARD HOME, *Bart. V. P. R. S.*

Read June 26, 1817.

THE nests of a particular species of swallow which is principally met with in the island of Java, have from time immemorial formed an article of trade between that island and China, where they are purchased at a high price by that voluptuous people, it being believed, that the materials of which the nests are composed, are possessed of an aphrodisiac virtue in an eminent degree. They have been occasionally brought into this country, and are preserved in collections of natural history, as curiosities. In what manner the bird procures the materials out of which the nest is made, has till now remained unknown; a thousand conjectures have, however, been made upon this subject. It has been supposed by some, that it is a gluten collected from the mollusca picked upon the surface of the sea. By others, a substance extracted from certain fuci found on the sea shore. By others again, a portion of the food in a half digested state regurgitated to be employed for this particular purpose. Sir STAMFORD RAFFLES, who has just returned from Java, where he resided five years, as lieutenant governor, has brought over a number of these nests, and has been kind enough to offer me some of them, for the purpose of investigating the nature of

the materials of which they are composed, and gives it decidedly as his own opinion, that, whatever it is, it is brought up from the stomach, and requires at times so great an effort, as to bring up blood, the stain of which is seen on the nest. This account of Sir STAMFORD RAFFLES, in the correctness of whose observation I have the greatest confidence, led me to investigate this subject, and to ascertain by examination whether this particular swallow has any glands that are peculiar to its œsophagus, or stomach, enabling it to secrete a mucus similar in its nature to the substance of which the nest is composed. I at the same time requested my friend, Professor BRANDE, to analyze one of the nests, and to inform me of its composition. In examining the gastric glands of the Java swallow, even with the assistance of a common magnifying glass, I saw an obvious difference between the appearance of the orifices by which the secretion is poured into the gizzard, and of those of other birds, but, as I had never examined those glands in the common swallow which migrates to this country, it became necessary, before I proceeded farther in the enquiry, to ascertain whether in all the swallow tribe there were similar structures. In the present season this opportunity has been afforded me, and I find that in the common swallow, both male and female, the orifices of the gastric glands differ in nothing from those of birds in general, but that the peculiar structure which I am about to describe is confined to the Java swallow. This bird, Sir STAMFORD RAFFLES informs me, does not migrate, but remains all the year an inhabitant of the caverns in that island. Some of the most extensive caves in which they reside, are forty miles from either sea. Those swallows that build their nests near the

sea, are observed to fly inland towards extensive swamps where gnats and other insects are in great abundance. Those that build in inland caves, are observed to quit the caves in the morning, and generally return in swarms darkening the air, towards the close of the day; they are, however, going in and out the whole of the day. This bird is double the size of our common swallow. There are two separate nests, one for the male to lie and rest in, which is oblong and narrow, adapted to his form, the other wide and deeper, to receive the female and the eggs.

As Mr. BAUER has been kind enough to make drawings of the gastric glands in the blackbird, the common swallow, and Java swallow, in which the parts are so much magnified, that the difference in their structure is obvious to the most superficial observer, it is not necessary in this place to enter much into detail respecting them: I shall only observe, that from what is represented in the drawings, it is evident that the gastric glands in the swallow tribe, both those that migrate and those that remain during the whole year in Java, do not afford the same supply of gastric liquor as in other birds, since they have a smaller receptacle belonging to the gland into which the secreted liquor is to be received. This circumstance confirms the observations that I made, upon a former occasion, respecting the gastric glands of the casuary of Java and of the ostrich, that these glands are largest in those birds that inhabit countries that afford a small supply of nourishment. The swallow of Java, as well as the casuary of that island, lives in perpetual plenty, and the swallow that migrates, although it travels from the equator to the pole, only remains in cold countries during the summer season,

while the sun is fertilizing them, and therefore has probably an equally abundant supply of its natural food in the regions of the north, as at the equator.

The only difference between the glands of the migrating swallow, and those of the blackbird, is the smallness of the reservoir, the surface of the gullet upon which the external openings of the glands are seen is exactly the same, there is not in the one or the other any apparatus for secreting mucus which is not common to birds in general.

In the Java swallow we have, on the other hand, a structure of a particular nature; there is a membranous tube surrounding the duct of each of the gastric glands, which, after projecting into the gullet for a little way, splits into separate portions like the petals of a flower: for what purpose so extraordinary an apparatus could be provided, would probably have puzzled the weak intellects of human beings, and many wild theories might have been formed respecting it, had not the animal matter of which the bird's nest is composed, and the accurate observation of Sir STAMFORD RAFFLES, who had no doubt that the materials of the nest were produced from the gullet, led to the discovery of its use.

That the mucus of which the nest is composed, is secreted from the surface of these membranous tubes, there is no more doubt than that the gastric juice is secreted from the glands whose ducts these tubes surround; and this confirms an opinion which I have adopted for many years, that membranes on which no glandular structure could be seen, were capable of secreting mucus; and now that we find those membranes, where their surfaces are so much magnified, exhibit no glandular structure, we may, without the chance of

more accurate observers refuting us, be satisfied that no such structure exists.

There are, perhaps, no more curious provisions given to animals by their Creator, than those which are to be employed for the preservation of their young, while it yet remains in the egg; but as many of these belong to the organs of generation themselves, or arise from secretions produced by glands immediately connected with them, they pass unnoticed, the mind being lost in the contemplation of so wonderful a contrivance as generation itself.

The present provision for forming a nest out of its own secretions, in an animal of so high an order as the class Aves, strikes us with astonishment, since birds in all other countries find substances of some kind or other out of which they form their nests, and makes it evident that this particular bird, at the time of its first creation, was intended to be the inhabitant of the caverns of Java, in which nothing is to be met with out of which a nest could be constructed, as the camel is adapted to the sandy deserts of which it is the natural inhabitant, both by the provision in its stomach for carrying a store of water, and the form of its hoof, which cannot, like that of other animals, be injured by walking in sand.

The swallows of Java that reside upon the coast, never exhaust their secretions in forming their nests when they find other materials fitted for that purpose.

The nearest approach to a provision of this kind, is in the insect tribe, the bee secreting wax out of which it forms its comb, both for the nest of its young, and a reservoir to contain supplies of nourishment.

The nest described in the preceding Paper, appears to con-

sist of a substance having properties intermediate between gelatine and albumen. It resists for a considerable time the action of warm water, but after some hours enlarges and softens; upon drying, it again resumes its former appearance and properties, becoming somewhat more brittle than before, probably, in consequence of having lost a very small portion of gelatine, which delicate tests discover in the water.

In the diluted acids, this substance dissolves with more ease than coagulated albumen; in the concentrated acids, its properties are nearly the same as those of coagulated white of egg.

With the caustic and subcarbonated fixed alcalis, it forms saponaceous compounds, which are decomposed by the acids with the same appearances as other albuminous soaps. It readily dissolves in liquid ammonia, and in the solution of subcarbonate of ammonia, circumstances in which it differs from albumen. When submitted to destructive distillation, a relatively small portion of ammonia is formed, and the remaining coal is easy of incineration, circumstances which likewise lead to point out a distinction between this substance and albumen.

Explanation of Plate XVI.

Fig. 1. The gizzard and part of the gullet of the Java swallow slit up and laid open, magnified twice in diameter, or 4 times in superficies.

Fig. 2. A small portion of the lowermost part of the gullet, magnified 15 times in diameter, or 225 times in superficies.

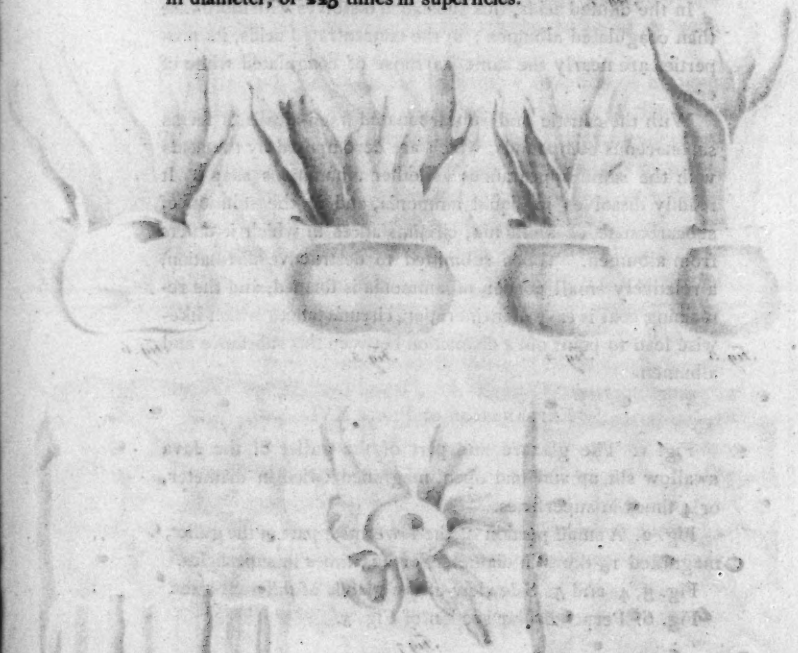
Fig. 3, 4, and 5. Side view of the glands of different sizes.

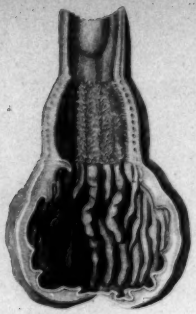
Fig. 6. Perpendicular section of Fig. 5.

Fig. 7. A top view of one of those glands; the figures 3, 4, 5, 6, and 7, are magnified 50 times in diameter, or 2500 in superficies.

Fig. 8. A small portion of the lowermost part of the gullet of the common swallow.

Fig. 9. A small portion of the lowermost part of the gullet of the black bird, both figures are magnified 15 times in diameter, or 225 times in superficies.





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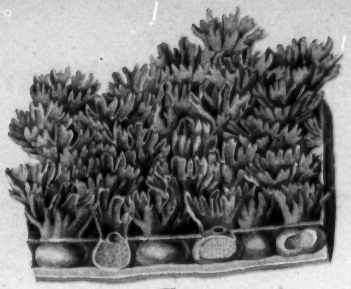


Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

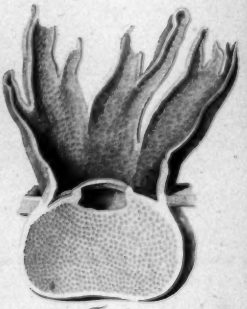
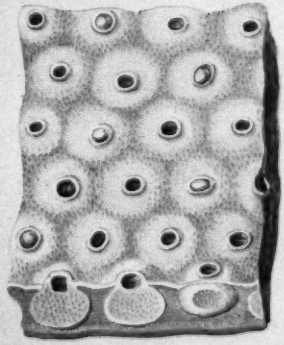


Fig. 6.



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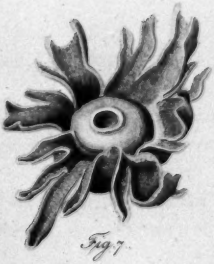


Fig. 7.

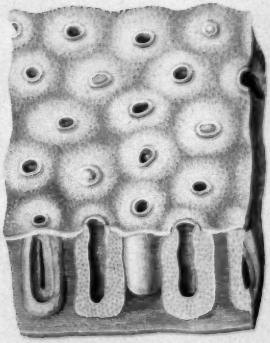


Fig. 9.

Franz Bauer. del.

J. Basire. sc.

XXVI. *Observations on the Hirudo complanata, and Hirudo stagnalis, now formed into a distinct genus under the name,* GLOSSOPORA. *By Dr.* Johnson, *of Bristol. Communicated by Sir* Everard Home, *Bart. V. P. R. S.*

Read June 26, 1817.

I BEG leave to lay before the Society, a few remarks on the *Hirudo complanata* and *Hirudo stagnalis;* animals that have been hitherto, but injudiciously, retained in the genus HIRUDO.

From the circumstance, in which they differ more particularly from the leech, they are now formed into a distinct genus, under the term of *Glossopora.*

These animals resemble the leech,

a. In the body being furnished with a series of rings.

b. In locomotion being effected by the alternate attachment of the head and tail.

c. In the division of one general stomach into several lateral cells or partitions.

These animals differ from the leech,

a. In the mouth being furnished with a projectile tubular tongue.

b. In the body being nearly flattened, and pyriform.

c. In having an abdominal pouch, or cavity, for the reception of their young.

The genus which they now form, takes the name (from γλῶσσα, a tongue, and πόρος, an aperture) of GLOSSOPORA.

CHARACTER GENERIS.

Corpus subovatum, depressum, caput acuminatum, lingua tubulata resiliens, os caudamque alterne affigendo progrediens.

G. tuberculata: *Glossopora dilatata, supra cinerea linea duplici tuberculata, subtus grisea, atomis nigris innumeris.*

LINNÆUS. *Syst. Nat.* XII. 2. p. 1079. n. 6.

——— *Faun Suec.* 2082, Hirudo depressa ovato-oblonga interaneis fuscis pinnatis pellucentibus.

HILL. *Hist. Anim.* p. 16. Hirudo lateribus attenuatis.

BERGMANN. *Act. Stockh.* 1757, Tab. 6. Fig. 12, 14. Hirudo sexoculata.

MULLER. *Hist. Verm.* 2. n. 157, p. 47. Hirudo complanata.

G. punctata: *Glossopora gracilis, cinereo-viridis, punctis plurimis subnigris.*

LINNÆUS. *Syst. Nat.* XII. 2. p. 1079. n. 5.

——— *Faun. Suec.* 2081. Hirudo (stagnalis) depressa nigra, abdomine subcinereo.

BERGMANN. *Act. Stockh.* 1757, n. 4. Tab. 6. Fig. 9—11. Hirudo binoculata.

MULLER. *Hist. Verm.* 2. n. 171. p. 41. Hirudo bioculata.

I am of opinion, that the *Hirudo circulans* described by Mr. SOWERBY,* and the *Hirudo crenata* by the Rev. WILLIAM KIRBY,† belong to this genus; but having had no opportunity of seeing either, I cannot determine whether or not they possess the tubular tongue.

* British Miscellany, Tab. 76. † Linn. Trans. vol. ii. p. 316.

The *Hirudo Hyalina* which, MULLER observes, has a flattened body, and carries its young in a pouch, and the *Hirudo tessulata* of the same author, will, I think, also be found upon examination, to belong to this new genus.

The tubular tongue very seldom falls within our view; hence our surprise is the less, that it should so long have escaped notice. At the time I first observed it, I was unacquainted with any author, who had mentioned it. On my referring, however, to BERGMANN'S account of the *Hirudo sexoculata*, (now *G. tuberculata*) I find it there noticed, not as a tongue, but as a slender body, of a whitish colour, occasionally projected from the mouth; of the use of which he confesses himself to be ignorant. I give his words, "*Utur munnen har jag åtskille gånger sett uträckas en blek, smal lem, hvars mytta är mig obekant.*"* MULLER mentions that he never witnessed (although he frequently looked for it) the body which BERGMANN saw the *Hirudo sexoculata* thrust from its mouth: but he once observed the *Hirudo vulgaris* protrude a similar organ, when, he says, this assertion of BERGMANN came across his mind. On his observing it, however, more narrowly, it proved to be a small aquatic worm that the animal had swallowed and afterwards rejected. This readily accounts, he adds, for the mistake into which BERGMANN has fallen.

In stepping forward to support BERGMANN, I am only doing an act of justice to the merits of an accurate and intelligent observer.

Having had the *G. tuberculata* and *G. punctata* under my

* Stockholm Transactions, for 1757, P. 313.

daily notice for a period of at least six months, I may, I presume, speak with some decision on this point. It may seem perhaps unnecessary to add, that I possess an elegant preparation, showing this tongue (protruded from the mouth) filled with mercury. It is of a cartilaginous structure, and admits of great flexibility. It is in length about one-eighth of an inch, and is seen, delineated (magnified) in Fig. 5. (Pl. XVII.)

The *G. tuberculata*, is about half an inch in length, but when fully extended, one inch. It is commonly found in rivulets, attached to pieces of wood, stones, &c. A delineation is given of its natural size, in a front and back view, in Fig. 1, 2, 3, 4. It possesses great transparency, and has a fine glossy vitreous appearance. It is convex above, flattened beneath, and somewhat resembles a compressed pear, the tail being very broad, and the head tapering towards the extremity, in which may be seen six eyes, (Fig. 9. *b*) disposed in two longitudinal rows. The sides or margin of the body are serrated. The back is usually of a brown colour, with lighter or darker patches, ornamented in the middle with a double longitudinal row of white tubercles. (Fig. 9. *c*.) These tubercles are connected together by two black longitudinal lines, and are seldom apparent, unless the animal is at rest. The belly is generally of one uniform colour, chiefly grey, with a slight double black line running longitudinally in the centre.

When this animal is in motion, (to which it is much averse, seldom quitting the spot on which it may be affixed,) it is observed to throw forward its head to the greatest point of extension, and then attach itself by means of the sucker ter-

minating that extremity. Thus securing its hold, it draws up the tail, when the back describes an arch, as in Fig. 6. The tail is then fixed by a similar sucker, and the head is again extended. Now and then it supports itself by the tail, the head waving to and fro, and occasionally buries its head under the abdomen, somewhat after the manner of an *oniscus*. This is seen in Fig. 7.

The *G. punctata* is much smaller than the above, and exceedingly delicate in its structure. On the head, we notice two eyes, placed transversely (Fig. 13. *b*). It is delineated of its natural size, in Fig. 11, 12. Both the back and belly is of a dusky grey, profusely sprinkled with minute black specks.

The stomach of the *G. tuberculata*, like that of the leech, is divided into several cells or partitions (Fig. 10. *d. e.*) with their extreme points verging towards the tail. The two last of these cells (*f*) are much longer than the rest, and terminate in two blind sacs. Between them, we notice the intestine, a tortuous tube extending to the anus, (Fig. 9. *e.*) a foramen above the rim of the circular sucker, or what constitutes the tail.

These animals, it has been asserted, when cut, or divided, are capable of reproduction, but this seems to rest on no just foundation.

Their food principally consists of the *water helices*, and here we see the great use of the projectile tubular tongue. The animal, from its tapering so much towards the head, is enabled to penetrate some considerable way into the hollow of the shell, and from the flexibility of its tongue, can follow its victim to the innermost recess of its habitation. A passage

from Muller, upon this subject, may not be devoid of interest. "Spectaculum singulare præbuit hujus (*G. punctata*, olim *H. bioculata*) cum limace *Planorbis* conflictus: limacem ore prehendere molitur hirudo; ille se quam citissime cum strepitu ex aeris et aquæ subitanea pressione orto testa condit. Hirudo oram aperturæ tentare pergit, at Limax insidias sentiens, seque in domuncula hunc contra hostem minus tutum credens, animum capit, egreditur et festinanter ad summum vasculi marginem prorependo ex aqua aufugit. Miratu dignus *Limacis* instinctus salutem quærendi fuga in elementum *Hirudini* contrarium;

> Omnibus ignotæ mortis timor, omnibus hostem
> Præsidiumque datum sentire, et noscere teli
> Vimque modumque sui.

Paucas tamen post horas, jubente natura, in aquam rursus descendere coactus, novo sese periculo obtulit, eique demum succubuit."

The *G. tuberculata*, and *G. punctata*, are oviparous; the former producing about 50, the latter about 20 at a birth. The same appearance is observed in these animals as in the *H. vulgaris*, when they deposit their ova, that is, a contraction of the body both above and below the abdominal foramen (Fig. 8.). There is, however, this difference; the *H. vulgaris* deposits its ova *in a capsule, formed exterior to the body*, whilst in these animals, the ova are simply excluded, held together by some gelatinous matter. From six to twelve ova are deposited at a time. When the whole of the ova are excluded, they are received into the abdominal pouch of the parent, where they constantly remain, until their contents are fully evolved. If the ova are removed, and kept in a vessel by

themselves, they do not prove productive; hence there seems to be a necessity for this parental solicitude. This pouch, or cavity, is always conspicuous in the *G. punctata*, but in the *G. tuberculata*, only at the time of its producing young. When the young are excluded from the ova, they remain attached to this cavity by the tail, enjoying a free extent of motion with the rest of the body. In this position they are represented in Fig. 7. They frequently leave this pouch, but soon return, and again affix themselves. Shortly after birth, their *interanea* are filled with a cream coloured fluid, which, under the microscope, presents a most interesting and beautiful appearance. Whilst speaking of the stomach of the parent animal, I forgot to observe, that the *interanea* are only visible when food has been recently taken. I mention this circumstance, that I may not be supposed to labour under a mistake, in the view I have given of this organ. MULLER himself indeed confesses, that he was a whole month (although assisted by the microscope) before he discovered it, yet he with much candour adds, that he afterwards very frequently saw it, even with the naked eye.

JAMES RAWLINS JOHNSON.

London, June 18, 1817.

EXPLANATION OF PLATE XVII.

Fig. 1, 2. The *G. tuberculata*, natural size, front view.

Fig. 3, 4. Ditto, back view.

Fig. 5. The tubular tongue (magnified): that portion from *a* to *b* is usually protruded from the mouth, the letter

c refers to the root or the expanded part of the tongue, or what, more properly speaking, constitutes the œsophagus.

Fig. 6. Shows the arched back the *G. tuberculata* presents, when in motion.

Fig. 7. Shows the abdominal pouch of this animal with the young affixed to it by their tail, enjoying free motion with the rest of the body.

Fig. 8. Its appearance when about to deposit its ova; *a*, the mouth; *b*, the contracted portion containing the ova to be then excluded; *c*, the remaining portion of the ova left in the abdominal cavity; *d*, the tail.

Fig. 9. The *G. tuberculata* magnified, showing, *a*, the tubular tongue; *b*, the eyes; *c*, the double longitudinal row of white tubercles; *d*, the serrated margin; *e*, the anus.

Fig. 10. The back part of the same animal, showing, *a*, the tubular tongue projected from the mouth; *b*, the œsophagus; *c*, the abdominal foramen; *d*, the alimentary canal or stomach; *e*, the lateral cells of the stomach; *f*, the two last long cells; *g*, the intestine; *h*, the sucker, or tail.

Fig. 11. *G. punctata*, natural size, front view.

Fig. 12. Ditto, back view.

Fig. 13. The same animal, magnified, showing, *a*, the projectile tubular tongue; *b*, the eyes; *c*, the abdominal pouch; *d*, the tail; *e*, the serrated margin.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

Fig. 11.

Fig. 12.

Fig. 13.

J. R. J. del.

J. Basire sc.

XXVII. *Observations on the gastric glands of the human stomach, and the contraction which takes place in that viscus.* By Sir Everard Home, *Bart. V. P. R. S.*

Read June 26, 1817.

In the year 1807, I laid before the Royal Society some observations on the human stomach, and I am now enabled, through the assistance of Mr. Bauer, to exhibit magnified views of the internal membrane of that viscus, in which the different structures composing its surface are distinctly shown.

The magnified drawings of the gastric glands of the Java swallow, so lately exhibited, must still rest upon the minds of those members who saw them; they are of so instructive a nature, that I was led to request Mr. Bauer would make similar representations of the glands of the lower part of the human œsophagus, and of the surface of the internal membrane of the stomach and duodenum.

The stomach employed for this purpose was under the most favourable circumstances, as the patient had died of an apoplexy, having no other bodily complaint.

The glands situated in the lining of the lower part of the œsophagus, which in my former Paper were called œsophageal glands, when examined in the microscope, have the appearance of infundibular cells, whose depth does not exceed the thickness of the membrane. This structure, however different from that of the gastric glands of birds, is a nearer

approach to it, than is to be met with on any part of the internal surface of the stomach or duodenum; it also resembles them in the secretion it produces coagulating milk, and none of the inspissated juices met with in these cavities affect milk in the same way. From these facts there can no longer be any doubt entertained, that the gastric glands have the same situation respecting the cavity of the stomach as in birds.

In my former investigation, the analogy of the bird would have led me to the same conclusion, had not the gastric glands of the beaver, which are more distinct than in any other quadruped, been a stumbling block in my way: but now the situation of these glands in the beaver and woombat, must be considered as an exception to the general rule, the necessary complexity of their structure making them too large to admit of their being conveniently placed, as is usual, in the œsophagus.

The structure upon the upper arch of the stomach, which, when magnified by a common lens, had the appearance of glands, is shown by Mr. BAUER to be made up of cells in the form of a honeycomb, the sides of which are not formed by doublings of the membrane, for no stretching of the cells alters the form of their orifices, but are regular partitions constructed between the cells. This honeycomb structure consists of cells of the greatest depth in this particular situation, but it is met with over the whole surface of the cardiac portion of the stomach, only the appearance is so faint as to require a great magnifying power to render it visible. In the pyloric portion the cells, in general, have the same appearance, but there are small clusters, the sides of which

rise above the surface, giving the appearance of foliated membranes. In the duodenum this takes place in a greater degree, and the loose edges of these membranes, when entangled in the mucus that covers them, puts on an appearance of rounded glandular bodies, but these admitted of being expanded so as to explain the deception.

The description which I have given of the internal membrane of the stomach, proves how nearly my late ingenious friend, Dr. GEORGE FORDYCE, who examined its surface with very inferior means to those employed upon the present occasion, had approached the truth, when he declared it to be composed of cellular membrane.

I have shown upon more occasions than one, that the gastric glands are both largest and most numerous in those animals destined to inhabit the least fertile regions of the earth, and are smallest as well as fewest where the supply of food is most abundant, to prevent the body being injured by the effects of over feeding. If this arrangement was necessary in animals, it became still more so in man, whose means of procuring and preparing food for himself so much exceed those of all other animals, and who is, contrary to his reason, too readily disposed to carry the indulgencies of the table to excess. In him the gastric glands, as it was natural to expect, are so small, as to require the aid of Mr. BAUER's microscope to prove that they belong to the same series of structures as the gastric glands of the ostrich, which admit of being minutely examined by the naked eye.

Much is still wanting to enable us to understand the process of digestion: it is, however, no small step in this investigation, that a more correct knowledge of the structure of the

cavities in which it takes place, has been acquired; from which we learn that there are three different kinds of organization employed, in adding to the food three different ingredients which are requisite for its conversion into a material that can be assimilated with living animal matter, and be employed in carrying on the functions of life, also supplying the waste which is constantly taking place. The most important of these is evidently the gastric glands, next in order may be considered the honeycomb structure, and least so, although by no means unnecessary, the foliated membranes, which we know, from what takes place in the Java swallow, form the mucus that is mixed with the other ingredients.

That the stomach is occasionally met with after death divided into two portions by a muscular contraction, I have shown upon a former occasion, and I have there given it as my opinion, that this takes place while the process of digestion is going on. This opinion may, in the minds of many physiologists, require some stronger proofs than I have been able to give, but this, like many other muscular contractions which take place during life, is so often removed in the very act of dying, as rarely (and in some of them never) to be seen after death. We are therefore indebted to the effect of disease sometimes rendering them permanent, for any knowledge we have of their existence.

In this way, strictures in the œsophagus, just where the fauces terminate, and the œsophagus begins, being of frequent occurrence, and upon examination of the parts after death, found to have taken place unconnected with any disease in the surrounding parts, teaches us, that this part has an involuntary contraction when any irritating matter is applied, and

thus forms a guard to prevent substances that would prove hurtful to the stomach from being swallowed.

Strictures in the urethra immediately behind the cavity of the bulb, being met with under the same circumstances after the use of irritating injections into that canal, and various other causes of irritation, is the only evidence we have of this part having a power of involuntary contraction, which in the act of the coitus is employed to prevent any part of the semen from being forced backwards into the bladder.

Through the kindness of Mr. CARPUE, I am now enabled to produce a specimen of permanent contraction in the stomach, and if I had not observed such a contraction before, this specimen would have led, as in the other cases just mentioned, to the discovery of the stomach in some of its natural actions, having this kind of contraction take place in it. It was met with in the body of a woman, and was probably the cause of her death, as no other appearance of disease was met with: the body was exceedingly emaciated, but there was no opportunity of acquiring any information of the symptoms under which she laboured while alive.

As in this instance the stomach could be distended freely without any risk of the contraction giving way, the line of partition between the cardiac and pyloric portions is exactly defined, and shown in the drawing not to be the casual contraction of a few of the transverse muscular fibres, which might have happened equally to any of the others, but the contraction of a part that had always been liable to it, and which was to answer some purpose in the performance of the functions of that viscus.

The importance of this fact in studying the physiology of

the stomach, is the only apology I shall make for having pressed it so much upon the attention of the Society. Its use in the pathology of that viscus, although perhaps of still more importance to the cause of suffering humanity, this is not the proper place to consider.

EXPLANATION OF PLATES XVIII, XIX, XX.

PLATE XVIII.

Fig. 1. $\frac{1}{100}$ parts of a square inch of the lower part of the œsophagus, magnified 15 times in diameter, or 225 times in superficies.

Fig. 2. $\frac{1}{100}$ parts of a square inch of the cardiac portion, magnified 15 times in diameter, or 225 times in superficies.

Fig. 3. $\frac{1}{100}$ parts of a square inch of the same cardiac portion, magnified 30 times in diameter, or 900 times in superficies.

PLATE XIX.

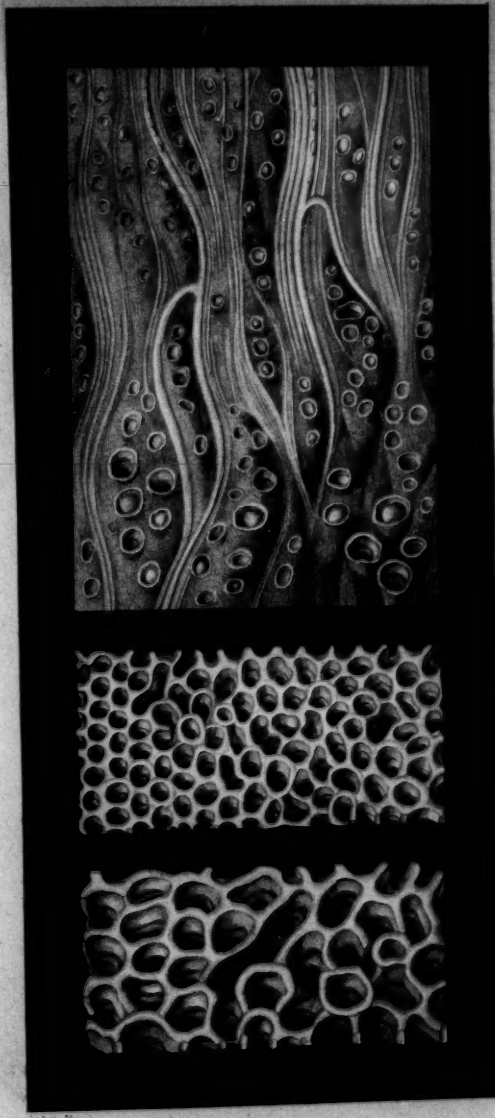
Fig. 1. $\frac{1}{100}$ parts of a square inch of the pyloric portion, magnified 15 times in diameter, or 225 times in superficies.

Fig. 2. $\frac{1}{100}$ parts of a square inch of the same pyloric portion, magnified 30 times in diameter, or 900 times in superficies.

Fig. 3. $\frac{1}{100}$ parts of a square inch of the duodenum, immediately joining the pylorus, magnified 15 times in diameter, or 225 times in superficies.

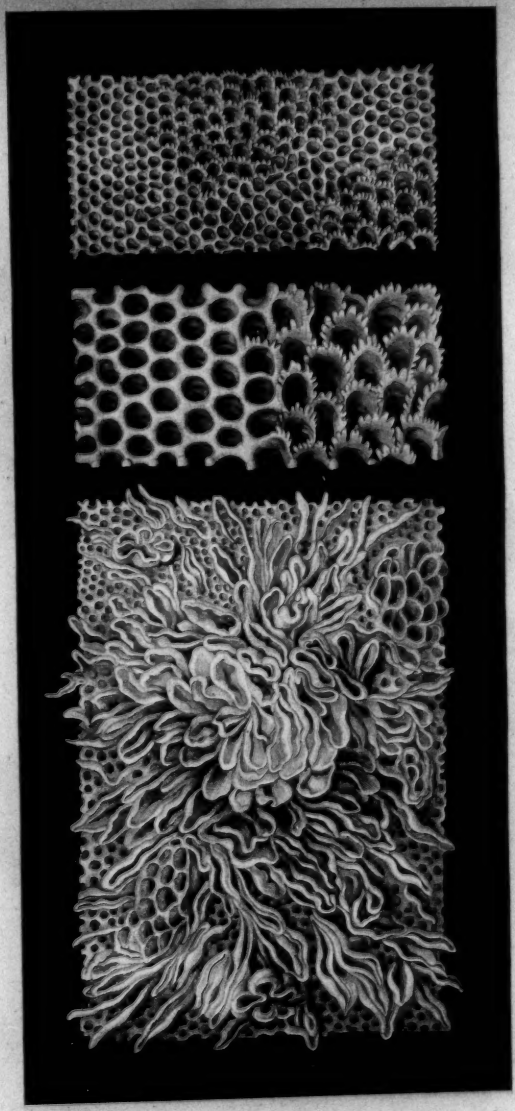
PLATE XX.

The human stomach, in a distended state, to show a permanent contraction, which had taken place in consequence of disease, separating the cardiac from the pyloric portions.



Franz Bauer. del.

J. Basire. sc.



Franz Bauer. del.

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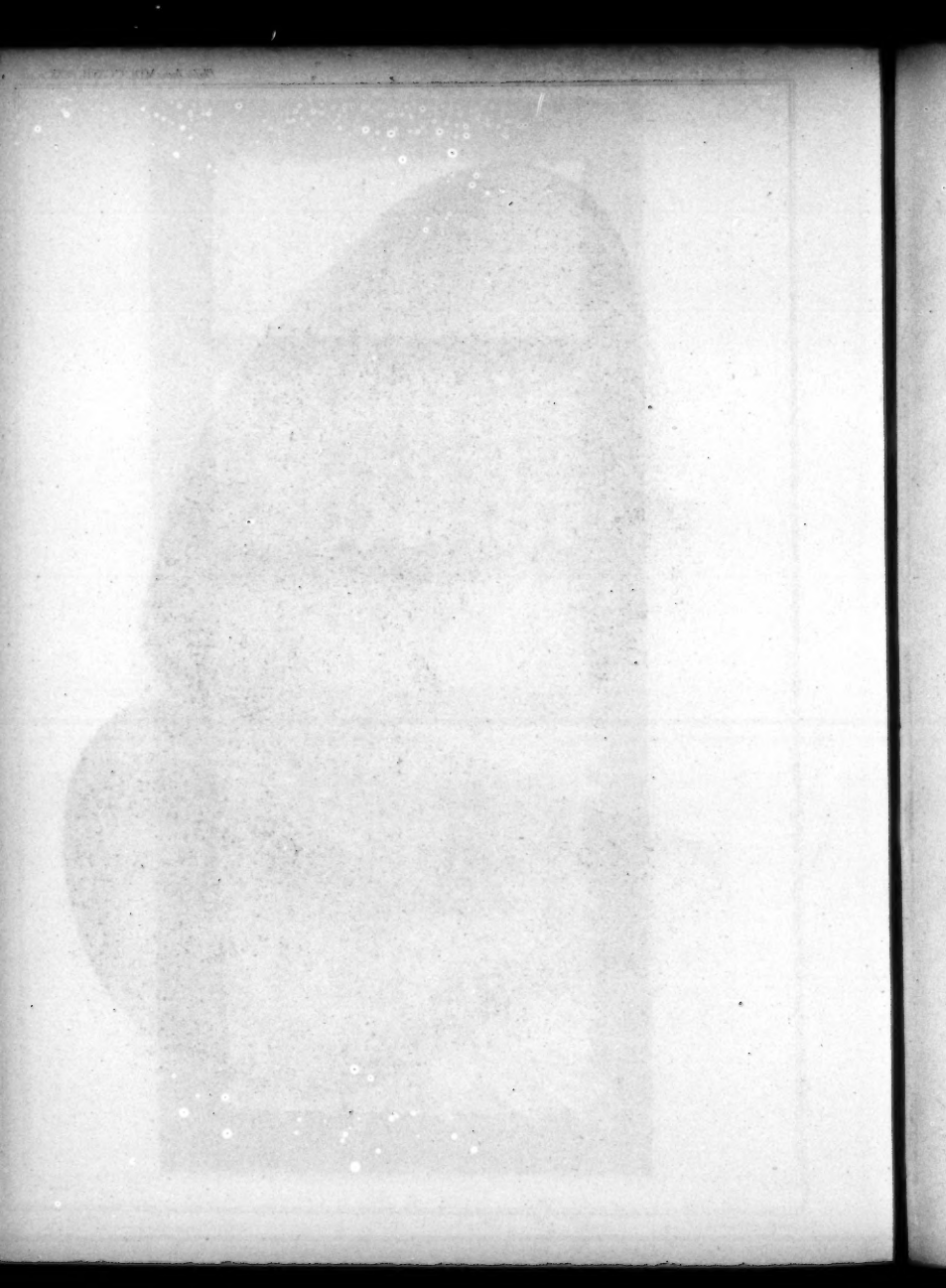

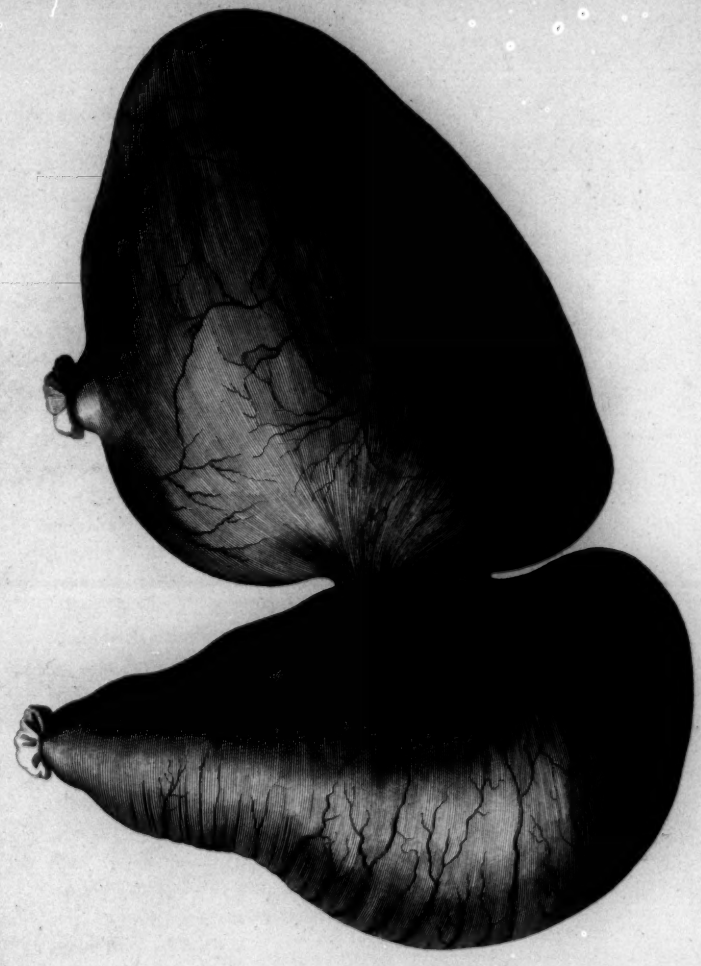

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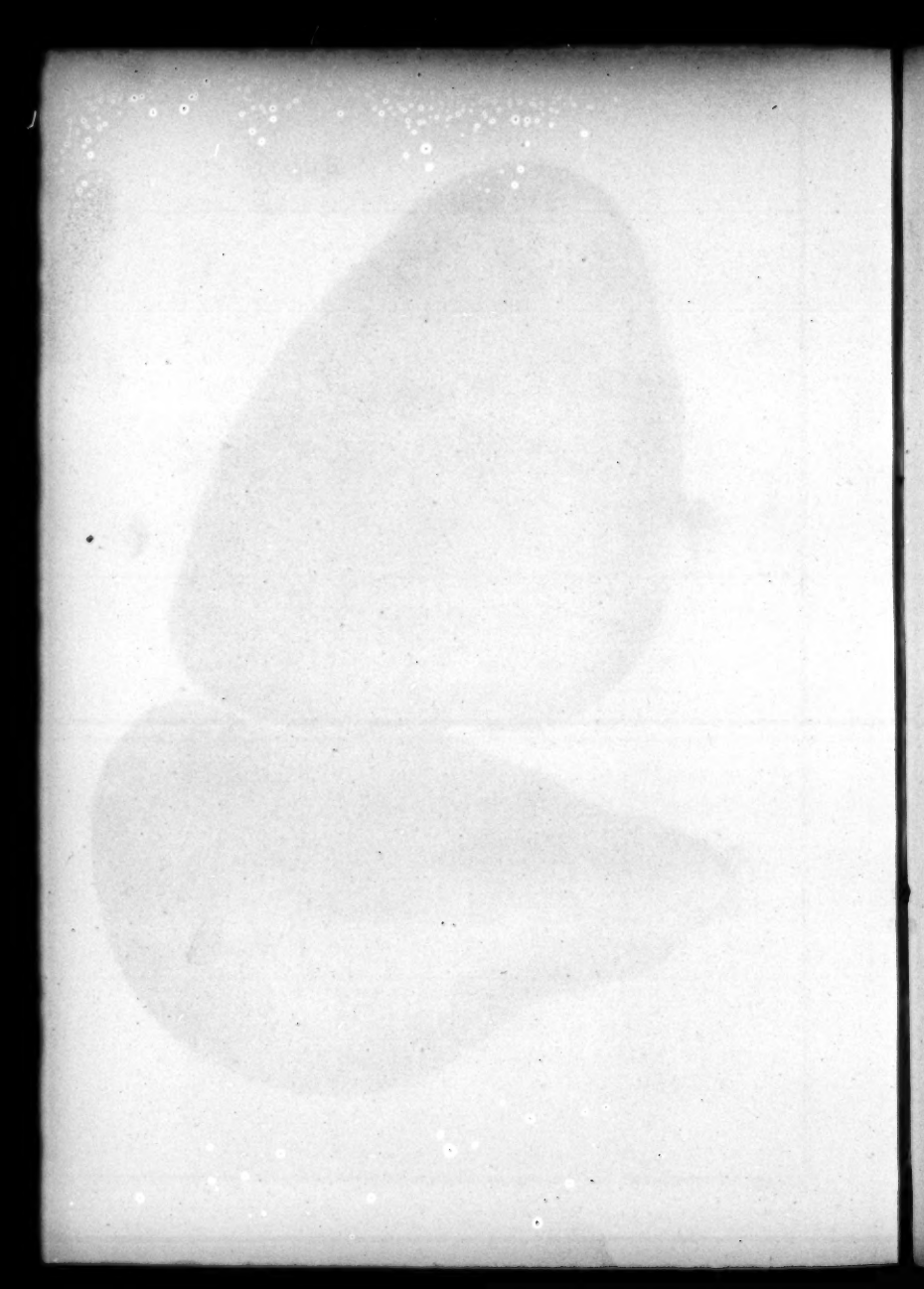

XXVIII. *On the parallax of the fixed stars.* By John Pond, Esq. *Astronomer Royal.*

Read June 26, 1817.

In a former Paper on the subject of parallax, I mentioned my intention of prosecuting this investigation by means of fixed telescopes on a new construction, and which I conceived were better adapted to this purpose, than any other instrument hitherto employed. One of these was directed to α Aquilæ, the other to α Cygni. Both instruments have answered the intended purpose. The observations of α Aquilæ were interrupted by some alterations which I found it necessary to make in the construction of the instrument; but those of α Cygni have been continued without intermission from the month of January to the present time.

Strictly speaking, a much longer period would be required (even with a perfect instrument) to determine either the exact quantity of parallax, in case it could be discovered, or the limit which it does not exceed, should it prove to be too small to be susceptible of observation; since the uncertainty in the annual variation of each star, bears a considerable proportion to the quantity to be investigated.

Notwithstanding, however, this very small uncertainty which future observation alone can remove, it appears to me that the observations already obtained, afford a proof sufficiently satisfactory, that the discordances which formed the subject of my last communication, must have had some other cause than parallax.

Though the method I have adopted in this investigation is extremely simple, I am not aware that it has been ever employed before. I do not attempt to refer the position of the star whose parallax is to be ascertained to any point determined by a level or plumb line, but by means of a micrometer within the telescope I measure only its difference of declination from some other star which passes through the same field; the telescope itself being securely fixed on a stone pier. The star chosen for this purpose should differ as much as possible in right ascension from the star whose parallax is to be investigated, that the results may be affected by the sum of the parallaxes of both stars. It should likewise be sufficiently bright to be visible even when it passes the meridian at noon, otherwise the observations must necessarily be subject to a period of interruption at the very time they would be of the most importance. Should the difference of declination of the two stars exceed 8 or 10 minutes, it will be advisable to apply a double micrometer to the telescope; that is, a micrometer having two moveable wires, such as have been made for some years past by Mr. TROUGHTON, and are, I believe, familiar to most astronomers of this country.*

The star I have selected is β Aurigæ, and the annexed observations from Jan. 21st, to May 16th, will show to what a very singular degree of precision results may be obtained.

* As Mr. TROUGHTON's numerous avocations could not have permitted him to satisfy my impatience, which made me very anxious not to lose a season, the instrument was made by Mr. DOLLOND. It was executed in great haste (being completed in less than three weeks); every part of it, nevertheless, is finished with such care and precision, that I regard it as a most valuable acquisition to the Observatory, and worthy to hold a place in a collection of instruments, which I believe cannot be equalled in Europe.

Result of 54 *observations of* α *Cygni compared with* β *Aurigæ, from* 21*st January* 1817, *to the* 16*th of May.*

α Cygni + β Aurigæ			Mean of each series of six Observations	Mean of each series of 18 Observations
Jan.	8″.197 +	9″.705 = 17″.902		
	8.295 +	9.673 = 17.968	8″.249 + 9″.757 = 18″.006	
	8.255 +	9.892 = 18.147		
	7.913 +	10.058 = 17.971		
	8.225 +	9.966 = 18.191	8.133 + 9.861 = 17.994	
	8.262 +	9.560 = 17.822		
	8.047 +	10.072 = 18.119		
May	7.553 +	10.536 = 18.089	7.604 + 10.473 = 18.077	
	7.213 +	10.812 = 18.025		

As it so happens that δ Cygni and *b* Aurigæ pass through the field of the telescope, they have likewise been observed.

Observations of these stars cannot be made directly useful in investigating the parallax of α Cygni, but may be of some importance in determining other equations; they likewise serve to show the limits of error to which the instrument may be liable.

The above observations do not include the extreme positions of the star in which it arrives at its maximum and minimum of parallax. As it appeared desirable to extend the period of observation as much as possible, I requested permission to insert the continuation of them to the time the impression of my Paper was required to complete the volume. They are therefore continued to the 21st of September, at which

time the summer observations may be presumed to terminate. Beyond this period, the observations rather belong to the autumnal or neutral state of parallax.

On the result of the observations thus continued, I beg leave to add the following remarks.

If the total number of observations be divided into three periods according to the state of the combined parallax, the result will appear to be as follows.

Winter Observations.	Rev.	″		Rev.	″
25 α Cygni	24 +	8.178	} their sum =	59 +	17.984
28 β Aurigæ	35 +	9.811			
Neutral	**Rev.**	**″**		**Rev**	**″**
26 α Cygni	24 +	7.920	} their sum	59 +	17.964
29 β Aurigæ	35 +	10.044			
Summer	**Rev.**	**″**		**Rev.**	**″**
32 α Cygni	24 +	3.340	} their sum	59 +	18.165
22 β Aurigæ	35 +	14.825			

The summer of this year has proved the most unfavourable for observation of any I recollect for several years. Many of the observations of β Aurigæ were extremely unsatisfactory. Notwithstanding this, I do not consider the small discordance of 0″.2 in the total distance of the two stars in a direction contrary to the effect of parallax as accidental, but trust that in time I shall discover the cause. For the present, however, I am willing to attribute it to error of observation. Now, even in this most unfavourable point of view, I think we may venture to infer, that (supposing the proportion of parallax of each star not to differ greatly from their degree of brightness) the mean place of either of them *is never deranged* by parallax *above one tenth of a second*, because such a deviation would produce, by its double effect on each star, a

total discordance of nearly 0″.8, which I really think exceeds the limits of my uncertainty.*

Should the parallax of α Cygni be admitted to be insensible, it would follow from the observations with the mural circle, that it was equally so in α Lyræ and γ Draconis, as these stars have assiduously been observed during a period of 5 or 6 years, without any perceptible difference at opposite seasons.

Of α Aquilæ, I cannot speak with the same confidence, but as I propose to make my observations of this star the subject of a separate communication, I need not now anticipate it, and shall only observe, that in a star so far from the Zenith, it would be rather unsafe to ascribe any small discordance to parallax, except confirmed in brighter stars more advantageously situated for observation.

TABLE I. Contains all the observations made with the instrument corrected for the usual equations, β Aurigæ is observed on the northern wire, the other three stars on the southern wire.

TABLE II. Contains the results arranged in three series.

TABLE III. Contains in a similar manner the results of α Cygni and δ Cygni, but the early observations of δ Cygni are not to be relied on, the star being scarcely visible from its vicinity to the sun, the uncertainty I think does not exceed 0″,25. It is evident that they both have the same parallax.

TABLES IV. and V. Give the coefficient of parallax, and require no explanation.

* This very small uncertainty may, I think, be somewhat diminished by the following consideration: the mean of all the observations in March, April, September, and October, should give the summer distance of these two stars under any theory either of parallax or annual motion. Now the mean of 50 observations in these months is 18″.15. The exact coincidence of this quantity with that found by actual observation, indicates that the small discordance above noticed is not accidental, but in its progress is something similar to annual variation.

In the case of the Pole star, I find an annual variation that cannot be deduced from any comparison of distant observations.

TABLE I.

1817.	β Aurigæ Revolutions 35 +	α Cygni Revolutions 24 +	δ Cygni Revolutions 12 +	b Aurigæ Revolutions 12 +	1817.	β Aurigæ Revolutions 35 +	α Cygni Revolutions 24 +	δ Cygni Revolutions 12 +	b Aurigæ Revolutions 12 +
	″	″	″	″		″	″	″	″
Jan. 21	9,53	8,36	1,51	—	Mar. 22	9 71	8,00	3,39	10,71
22	—	—	—	—	23	—	8,50	—	—
23	9,65	—	—	—	24	—	—	—	—
24	—	—	—	—	25	10,25	—	—	11,15
25	—	—	—	—	26	—	8,13	—	—
26	—	—	—	—	27	9,73	—	—	—
27	—	—	—	—	28	—	—	—	—
28	—	—	—	—	29	—	8,55	3,65	—
29	—	—	—	—	30	—	—	—	—
30	9 84	8,44	—	—	31	9,72	8,07	3,54	—
31	9,87	8,18	1,85	—	Apr. 1	8,95	—	—	—
Feb. 1	8,79	—	—	—	2	9,27	8,59	4,53	—
2	—	—	—	—	3	9,14	8,53	4,07	—
3	—	—	—	—	4	9,70	—	—	—
4	—	8,25	1,15	—	5	9,57	8,33	3,24	—
5	—	—	—	—	6	—	—	—	—
6	9,53	8,19	—	—	7	9,48	—	—	—
7	—	7,76	—	—	8	9,52	—	—	—
8	9,81	—	—	—	9	—	7,84	—	—
9	—	—	—	—	10	11,24	8,21	—	—
10	—	—	—	—	11	10,30	—	—	—
11	9,57	—	—	9,70	12	—	—	—	—
12	—	—	—	—	13	—	—	—	—
13	—	8,15	—	—	14	9,95	—	—	—
14	9,72	—	—	10,86	15	10,10	—	—	—
15	9,61	8,24	—	10,49	16	10,45	7,55	—	—
16	9,54	—	—	10,34	17	—	8,25	—	—
17	—	—	—	—	18	—	7,67	—	—
18	8,75	8,74	—	—	19	10,38	8,25	—	—
19	9,72	7,27	—	—	20	9 80	8,01	—	—
20	—	—	—	—	21	10,71	—	—	—
21	10,26	8,51	—	—	22	8,95	—	—	—
22	9,85	—	—	10,24	23	10,24	8,55	—	—
23	—	8,33	—	—	24	—	—	—	—
24	10,04	7,80	2,83	—	25	—	—	—	—
25	10,02	8,36	2,55	—	26	—	8,57	—	—
26	—	8,28	—	10,94	27	—	—	—	—
27	9,54	—	—	10,67	28	—	—	—	—
28	9,68	8,31	—	10,61	29	—	—	—	—
Mar. 1	—	8,00	—	—	30	10,49	—	—	—
2	—	—	2,38	—	May 1	—	7,74	—	—
3	—	8,53	2,67	—	2	—	7,48	—	—
4	10,06	—	2,88	9,66	3	—	—	—	—
5	10,16	8,00	—	9,60	4	11,13	7,66	—	—
6	10,16	7,75	3,22	9,98	5	9,21	8,11	—	—
7	10,27	—	—	—	6	10,80	7,15	—	—
8	9,91	7,25	—	—	7	10,05	—	—	—
9	10,46	7,56	—	10,15	8	—	—	—	—
10	—	8,04	3,82	—	9	—	—	—	—
11	—	8,43	—	—	10	—	7,14	—	—
12	—	—	—	—	11	—	7,45	—	—
13	—	8,45	3,66	—	12	—	7,26	—	—
14	9,49	—	—	10,51	13	—	7,45	—	—
15	—	—	—	—	14	13,00	7,04	—	—
16	10,08	—	—	10,35	15	10,18	—	—	—
17	10,06	8,41	2,96	10,00	16	11,03	—	—	—
18	9,54	—	—	10,55	17	—	—	—	—
19	—	—	—	10,12	18	—	—	—	—
20	—	—	—	—	19	—	—	—	—
21	10,39	7,76	1,94	9,93	20	—	—	—	—

TABLE I.

1817.	β Aurigæ Revolutions 35 +	α Cygni Revolutions 24 +	ϑ Cygni Revolutions 12 +	b Aurigæ Revolutions 12 +	1817.	β Aurigæ Revolution 35 +	α Cygni Revolutions 12 +	ϑ Cygni Revolutions 12 +	b Aurigæ Revolutions 12 +
	″	″	″			″	″	″	
May 21	—	—	—		July 21	—	—	58,79	
22	—	—	—		22	—	3,15	—	
23	—	6,87	—		23	—	3,34	58,77	
24	—	—	—		24	—	2,73	57,97	
25	—	—	—		25	—	2,98	58,02	
26	—	7,14	—		26	—	—	57,76	
27	—	5,84	—		27	—	2,97	58.14	
28	—	—	—		28	—	2,64	57,61	
29	—	—	—		29	—	—	—	
30	—	6,67	—		30	—	3,26	58,08	
31	—	6,35	—		31	14,65	2,98	57,78	
June 1	—	—	—		Aug. 1	14,34	3,41	58,11	
2	—	—	—		2	—	—	—	
3	—	—	—		3	—	3,17	58,70	
4	—	5,97	—		4	—	—	—	
5	—	—	—		5	15,16	3,57	58,42	
6	—	—	—		6	—	3,97	58,19	
7	—	—	—		7	—	—	—	
8	—	—	—		8	15,67	3,27	57,75	
9	—	—	—		9	—	3,34	57,62	
10	—	5,50	—		10	—	2,81	57,91	
11	—	—	—		11	—	—	—	
12	—	—	—		12	—	—	57,66	
13	—	—	—		13	—	—	—	
14	—	5,78	—		14	—	3,78	—	
15	11,03	5,02	—		15	—	3,28	57,53	
16	10,77	—	—		16	16,34	—	—	
17	—	—	0,23		17	14,63	3,18	58,03	
18†	10,50	6,32	0,97		18	—	—	—	
19	12,98	6,73	0,72		19	—	3,89	—	
20		6,30	0,96		20	—	3,31	57,37	
21	13,11	6,47	1,46		21	14,96	2,36	57,22	
22	—	5,84	0,66		22	14,76	2,83	—	
23	—	—	—		23	—	—	57,51	
24	—	5,39	0,10		24	—	3,61	58,03	
25	—	4,68	59,40		25	14,25	2,70	57,42	
26	—	—	—		26	15,12	—	—	
27	—	—	—		27	14,54	—	57,73	
28	—	3,64	58,23		28	14,64	—	—	
29	—	—	—		29	15,55	4,11	58,15	
30	—	3,13	57,72		30	14,58	—	—	
July 1	—	—	—		31	15,09	3,00	57,91	
2	—	3,57	—		Sept. 1	15,46	3,03	57,93	
3	—	—	—		2	14,24	3,84	—	
4	—	3,66	57,55		3	14,77	—	58,90	
5	—	3,50	—		4	14,59	4,00	58,70	
6	—	3,66	58,07		5	14,60	3,55	58,40	
7	—	3,83	—		6	—	3,86	58,42	
8	—	4,33	58,22		7	—	3,75	58,76	
9	—	4,15	58,38		8	—	3,47	58,16	
10	—	3,26	58,52		9	—	—	—	
11	—	—	58,32		10	—	3,14	58,25	
12	—	—	57,83		11	—	3,22	58,28	
13	—	3,38	—		12	—	—	—	
14	—	—	—		13	—	—	—	
15	—	—	—		14	—	3,13	—	
16	—	3,58	59,29		15	—	—	—	
17	—	—	—		16	—	—	—	
18	—	—	—		17	14,58	—	—	
19	—	3,79	58,00		19	15,25	2,53	57,03	
					21	15,04	2,81	58,12	

† These observations I consider useless, the star being too near the sun. It was not quite invisible, but was less than the diameter of the wire. This may be remedied another year by the substitution of a cobweb. About this time, likewise, the instrument was much deranged by extreme heat.

TABLE II.

Jan. 21, to March 18.		March 21, to May 16.		July 30, to Sept. 21.	
Winter.		Neutral.		Summer.	
α Cygni	β Aurigæ	α Cygni	β Aurigæ	α Cygni	β Aurigæ
″	″	″	″	″	″
	9,53		10,39	3,26	
8,36	9,65	7,76	9,74	2,98	
8,44	9,84	8,00	10,25	3,41	
8,18	9,87	8,50	9,73	3,17	
8,25	9,53	8,13	9,72	3,57	
8,19	9,81	8,55	8,95	3,97	14,65
7,76	9,57	8,07	9,27	3,27	14,34
8,15	9,72	8,59	9,14	3,34	15,16
8,24	9,61	8,53	9,70	2,81	15,67
8,74	9,54	8,33	9,57	3,78	14,63
8,51	8,75	7,84	9,48	3,28	14,96
8,33	9,72	8,21	9,52	3,18	14,76
7,80	10,26	7,55	10,30	3,89	14,25
8,36	9,85	8,25	9,95	3,31	15,12
8,28	10,04	8,01	10,10	3,36	14,54
8,31	10,02	8,55	10,45	2,83	14,64
8,00	9,54	8,57	10,38	3,61	15,55
8,53	9,68	7,74	9,89	2,70	14,58
8,00	10,06	7,48	10,71	4,11	15,09
,75	10,16	7,66	8,95	3,00	15,46
7,25	10,16	8,11	10,24	3,03	14,24
7,56	10,27	7,15	10,49	3,84	14,77
8,04	9,91	7,14	11,13	4,00	14,59
8,43	10,46	7,45	9,21	3,55	14,60
8,45	9,49	7,26	10,80	3,86	14,58
8,41	10,08	7,45	11,05	3,75	15,25
	10,06	7,04	11,00	3,47	15,04
	9,54		10,28	3,14	
			11,03	3,22	
				3,13	
				2,53	
				2,81	
Mean Result		Mean Result		Mean Result	
″8.173 + ″9.811		″7.920 + ″10.044		″3.340 + ″14,825	
=59.Rev. + ″17.984.		= ″17.964.		= ″18,165	

TABLE III.

Winter.		Summer.			
α Cygni	ϑ Cygni	α Cygni	ϑ Cygni	α Cygni	ϑ Cygni
″ 8,33	′	″ 6,32	″ 0,97	″ 3,89	″ 58,03
7,70		6,73	0,72	3,31	57,37
8,36	1,51	6,30	0,96	2,36	57,22
8,28	1,85	6,47	1,46	2,83	57,51
8,31	1,15	5,84	0,66	3,61	58,03
8,00	2,83	5,39	0,10	2,70	57,42
8,53	2,55	4,68	59,40	4,11	57,73
8,00	2,38	3,64	58,23	3,00	58,15
7,75	2,67	3,13	57,72	3,03	57,91
7,25	2,88	3,66	67,55	3,84	57,93
7,56	3,22	3,66	58,07	4,00	58,90
8,04	3,82	4,33	58,22	3,55	58,70
8,43	3,66	4,15	58,38	3,86	58,40
8,45	2,96	3,26	58,52	3,75	58,42
8,41	1,94	3,58	59,29	3,47	58,76
7,76	3,39	3,79	58,00	3,14	58,16
8,00	3,65	3,15	58,79	3,22	58,25
8,50	3,54	3,34	58,77	3,13	58,28
8,13	4,53	2,73	57,97	2,53	57,03
8,55	4,07	2,98	58,02	2,81	58,12
8,07	3,24	2,97	57,76	3,97	58,42
8,59		2,64	58,14	3,27	58,19
8,53		3,26	57,61	3,34	57,75
8,33		2,98	58,08	2,81	57,62
		3,41	57,78	3,78	57,91
		3,17	58,11	3,28	57,66
		3,57	58,70	3,18	57,53
Mean Result		Mean Result			
″8.165 — ″2 939		″3.683—″58.396.			
=″12 Rev. + ″5.226.		=″12 Rev. + ″5.287.			

The winter observations of ϑ Cygni are very far from satisfactory; the image of the star, from its vicinity to the sun, was smaller than the diameter of the micrometer wire, and it could not therefore be accurately bisected.

TABLES IV. and V.

TABLE IV. *Showing the effect of parallax on α Cygni.*

21 June	— 0,78 +	21 Dec.
1 July	0,85	1 Jan.
11	0,88	11
21	0,90	21
1 Aug.	0,88	1 Feb.
11	0,84	11
21	0,77	21
1 Sep.	0,68	1 Mar.
11	0,57	11
21	0,43	21
1 Oct.	0,29	1 April
11	— 0,14 +	11
21	+ 0.01 —	21
1 Nov.	0,17	1 May
11	0,32	11
21	0,46	21
1 Dec.	0,58	1 June
11	0,70	11
21	+ 0,78 —	21

TABLE V. *Showing the effect of the combined parallax of α Cygni and β Aurigæ.*

21 June	1,15	21 Dec.
1 July	1,21	1 Jan.
11	1,33	11
21	1,20	21
1 Aug.	1,13	1 Feb.
11	1,05	11
21	0,92	21
1 Sep.	0,77	1 Mar.
11	0,60	11
21	0,40	21
1 Oct.	— 0,20 +	1 April
11	0,00	11
21	+ 0,22 —	21
1 Nov.	0,41	1 May
11	0,60	11
21	0,77	21
1 Dec.	0,92	1 June
11	1,07	11
21	1,15	21

In the above Tables the semi-annual parallax of each star is supposed equal to one second.

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